

Interactive I.T. Student Activity Sheets

Leaving Certificate Strand 3



- Student Activities written to match the I.T. interactive modules on the Project Maths Leaving Certificate Student's CD Strand 3
- Interactive Activity Sheets included to enhance students' understanding of mathematical concepts
- Simple and clear guidelines are provided to facilitate learning
- Interesting questions are provided to lead students to explore, construct and consolidate their learning

Preface

The NCCA have pointed out particular Key Skills in their Draft Syllabus. “While particular emphasis is placed in mathematics on the development and use of information processing, logical thinking and problem-solving skills, the new approach being adopted in the teaching and learning of mathematics will also give prominence to students being able to develop their skills in communicating and working with others. By adopting a variety of approaches and strategies for solving problems in mathematics, students will develop their self-confidence and personal effectiveness.” To help our students to adapt to and take advantage of this new spirit of the syllabus, we have produced Interactive I.T. Student Activity Sheets which incorporate an innovative and diversified learning environment for mathematics.

As we all know, the advancement in technology has changed the way we can learn mathematics. Therefore we have developed a number of interactive modules on our student’s CD to match this new development. With the help of these interactive modules, students can not only enhance their understanding in mathematics, but they can also enjoy learning it.

In order to help our students use the I.T. tools more effectively, ***Interactive I.T. Student Activity Sheets Leaving Certificate Strand 3*** are produced in this booklet. A student activity sheet is designed for the majority of the interactive modules on the CD. All student activity sheets provide simple and clear guidelines including:

1. Reference to the related topics in ***Project Maths Student’s Leaving Certificate Strand 3 section***
2. Purpose of the I.T. tools
3. Instructions for using the I.T. tools.

These Student Activity Sheets, which include many interesting questions, will lead students to explore, construct, and consolidate their knowledge of mathematics on their own with ease. We believe that with the help of these activities, students’ knowledge and understanding of mathematics will grow.

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Instructions for use

This booklet contains student activities to accompany the majority of the interactive files on the Leaving Certificate Strand 4 section of the student disk. The specific section of the course that the activity relates to is specified in the name of the activity. At the top of each student activity the students are told what interactive file on the student disk is to accompany the student activity.

Technical Problems

The student disk has a link situated on the left hand side of its front page called “Troubleshooting” this section gives instructions, if any of the following problems are encountered:

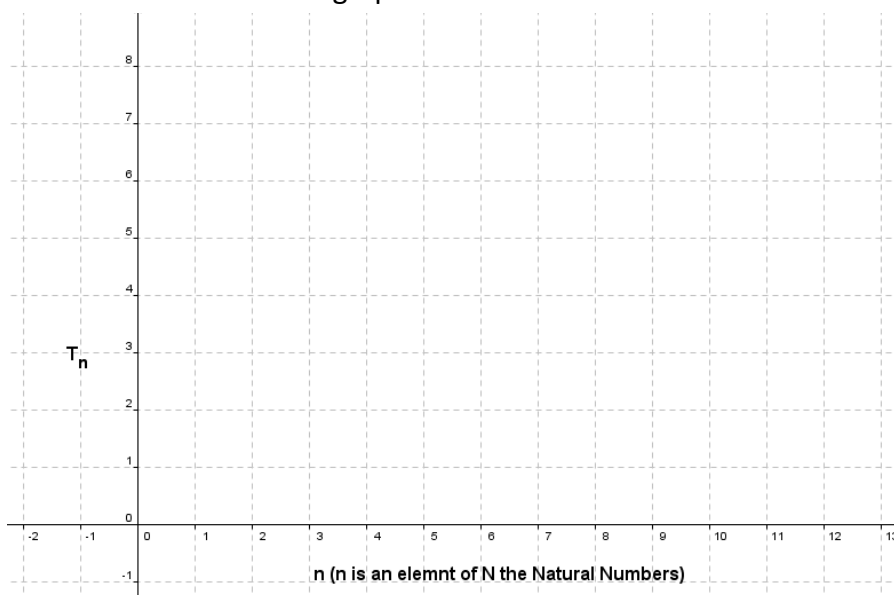
- Problems opening Office 2007 documents
- You do not have Java on your machine
- You do not have a PDF reader on your machine.

Student Activity: To investigate arithmetic sequences and series

Use in connection with the interactive file, 'Arithmetic Sequence and Series', on the Student's CD.

1. The first term " a " of an arithmetic sequence is equal to 3 and the common difference " d " of the sequence is 2.
 - a. Write down the first 6 terms. Show your calculations.

- b. Represent all these terms on a graph.



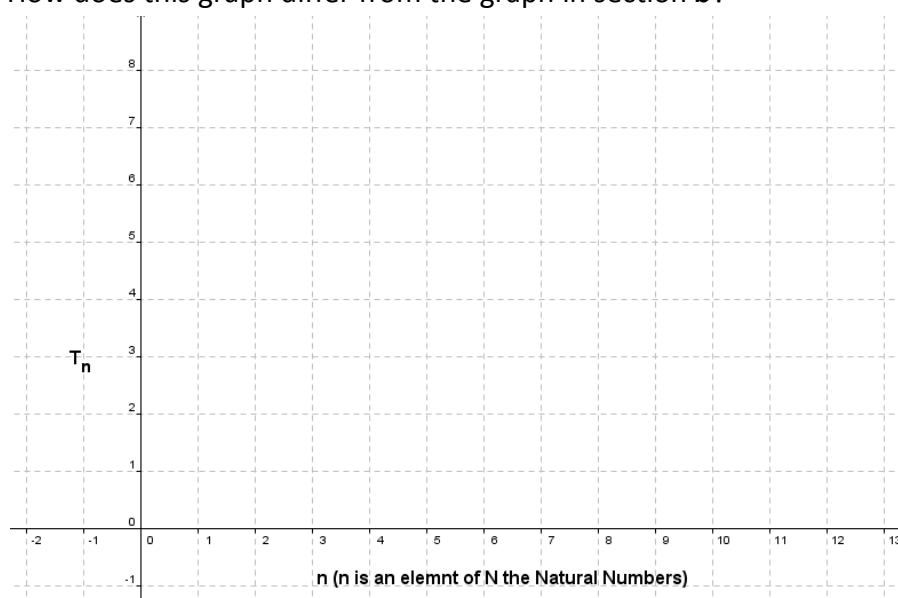
- c. Use any 2 points on the graph to find the slope of the line joining these two points.

- d. Repeat step c. for 3 more sets of points.

- e. What do you notice about the relationship between the slope and the common difference?

- f. Change the first term in the interactive file and keep the common difference as 2. What do you notice about this graph?

- g. The first term “a” of an arithmetic sequence is equal to 2 and the common difference “d” of the sequence is 2. Draw the graph to represent this situation. How does this graph differ from the graph in section **b**?

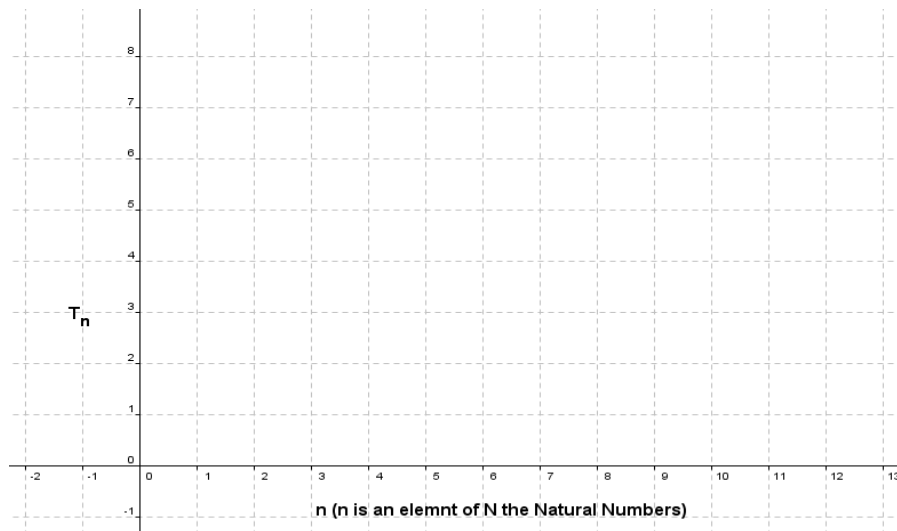


- h. What quantity do you need to change in the sequence to change the slope of its graph?

2. The first term “a” of an arithmetic sequence is equal to 5 and the common difference “d” is 2.

- a. Write down the first 6 terms. Show your calculations.

- b. Represent all these terms on a graph.

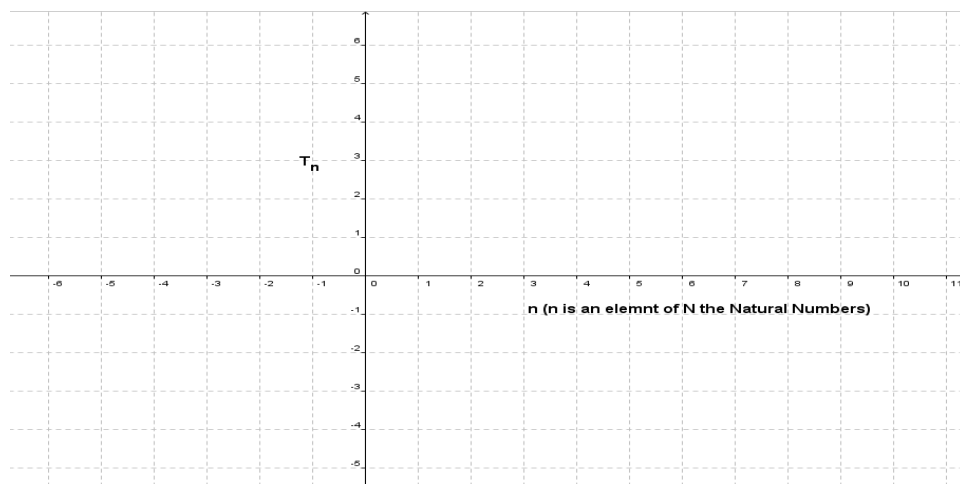


- c. What do you think the slope of the line joining these points will be? Check your answer using any 2 of the points on the graph. Explain the reason for this.

3. The first term “a” of an arithmetic sequence is equal to 6 and the common difference “d” is -1 .

- a. Write down the first 6 terms. Show your calculations.

- b. Represent all these terms on a graph.



- c. What do you think the slope of the line joining these points will be? Check your answer using any 2 of the points on the graph. Explain the reason for this.

4. The first term of an arithmetic sequence is 12 and the common difference is 5.

- a. Calculate the first 6 terms of this sequence. Show calculations.

- b. Calculate the sum of the first 6 terms of this sequence. Show calculations.

- c. Calculate the sum of the first 10 terms of this sequence. Show calculations.

- d. Complete the following table:

T_1	T_2	T_3	T_4	T_5	T_6

- e. Calculate $T_1 + T_6$.

- f. Calculate $T_2 + T_5$.

- g. Calculate $T_3 + T_4$.

- h. Use the pattern you have seen in the last 3 sections of this problem to find the sum of the first 6 terms

- i. Compare your results with that found in the interactive file.

5.

Given that the sum of the first n terms of an arithmetic sequence $S_n = \frac{n}{2}[2a + (n-1)d]$ and

the n^{th} term of an arithmetic sequence $T_n = a + (n-1)d$.

Show that $S_n = \frac{n}{2}[\text{first term} + n^{\text{th}} \text{term}]$.

6. After knee surgery, your trainer tells you to return to your jogging program slowly. He suggests jogging for 10 minutes the day after your treatment. Each day thereafter, he suggests that you increase that time by 2 minutes.

a. Is this an arithmetic sequence? Explain your answer.

b. What is the first term?

c. What is the common difference?

d. What are your jogging times for the first 6 days after your visit to the trainer?

e. For how many minutes will you jog on the 9th day after your treatment? Show calculations.

f. What is the total time you will jog in the first 9 days after your treatment? Show calculations.

g. Check your results for the previous 3 sections of this question using the interactive file.

- h. If you started jogging on the first day of the month, how long will you be jogging per day after 1 month (30 days)? Show your calculations.
-
-
- i. How much time will you spend jogging in the first month (30 days)?
-
-
7. A water tank contains 200 litres of water when full. The tank develops a drip and water leaks out at a constant rate of 5 litres per day.
- Does the amount of water left in the tank at the end of each day form an arithmetic sequence? Why?

- What is the first term?

- What is the common difference?

- How much water does the tank contain for each of the first 6 days?

- What is the total amount of water lost by the tank in the first 6 days?

- How many litres of water does the tank contain at the end of the 12th day?

- On which day will the tank be half empty?

- On which day will the tank be completely empty?

Arithmetic Sequence Quiz

1. What is an arithmetic sequence?
 - A. It is when you add the same value to the previous term each time to get the next term.
 - B. It is when you multiply the previous term by the same value each time.
2. What is the common difference?
 - A. It is the value you multiply the previous term by.
 - B. It is the value you add to the previous term.
3. Assuming the pattern continues, is 6,10,14,18 an arithmetic sequence?
 - A. No
 - B. Yes
4. What is the value of the 51st term of the following arithmetic sequence 2,7,12,17?
 - A. 252
 - B. 257
5. Assuming the pattern continues is the sequence 2,4,8,16 an arithmetic sequence?
 - A. Yes
 - B. No
6. Assuming the pattern continues is the sequence 8,4,0,-4 an arithmetic sequence?
 - A. Yes
 - B. No
7. Assuming the pattern continues is the sequence -4,-7,-10,-13 an arithmetic sequence?
 - A. Yes
 - B. No
8. A 14 year old girl has 100 teddy bears and does not want to part with them all at once so she decides to give away 2 teddy bears each birthday after this. What is the n th term of this sequence?
 - A. $100 + (n - 1) - 2$
 - B. $100 + (n - 1)(-2)$
9. Using the answer you got in the previous question, what is the term number when she has given away all of her teddy bears?
 - A. 50
 - B. 51
 - C. 64

10. Using the answer you got in the previous question, how old will the girl be when she has given away all of her teddy bears?
 - A. 51
 - B. 63
 - C. 64
11. Is the following sequence an arithmetic sequence 2, 4, 7, 11, 16?
 - A. Yes
 - B. No
12. Is $T_n - T_{n-1} = T_{n+1} - T_n$ always true for arithmetic sequences?
 - A. Yes
 - B. No
13. What effect does a negative common difference have on the terms of an arithmetic sequence?
 - A. Consecutive terms get bigger
 - B. Consecutive terms get smaller
14. If the first term of an arithmetic sequence is 1000, does that always mean that all the terms in the sequence are greater than 1000?
 - A. Yes
 - B. No
15. Is there a limit to the number of terms in the follow arithmetic sequence 10, 8, 6, 4, 2, 0, ..., ?
 - A. No
 - B. Yes
16. If the temperature in Ballybofey during the month of June increased by 0.5 degrees each day and it reached its peak at 25 degrees on the last day of the month, what was the temperature on the 1st of June in Ballybofey that year?
 - A. 10.5
 - B. 12.5
17. On the 17th July Martin has €400,000 savings and decides to spend €20,000 each year on the anniversary of this date, after how many withdrawals will he have €8,000 left? He ignores any interest he receives.
 - A. 16
 - B. 17
18. If an arithmetic sequence has a positive first term, how can subsequent terms become negative?
 - A. When the common difference is positive but very small.
 - B. When the common difference is negative.

19. Is an arithmetic sequence another name for an arithmetic progression (AP)?
- A. Yes
 - B. No
20. Could -99, -66, -33 be an arithmetic sequence and what will the 10th term be?
- A. Yes and the 10th term will be 199
 - B. No
 - C. Yes and the 10th term will be 198
21. An arithmetic sequence is an increasing sequence if and only if
- A. The common difference is very large and positive.
 - B. The common difference is positive.
 - C. The common difference is an integer.
 - D. The common difference is negative.
22. The 11th term of an arithmetic sequence is equal to 54 and the 7th term is equal to 34. Find a and d .
- A. $a=4$ and $d=5$
 - B. $a=-1$ and $d=5$
23. The first term of an arithmetic sequence is 1.1 and the 21st term of the same sequence is 41.1, find the 16th term.
- A. 30
 - B. 31.1
 - C. 32
24. Does the sequence $\{a-3d, a-2d, a-d, a, \dots\}$ form an arithmetic sequence?
- A. No
 - B. Yes
25. The sixth term of an arithmetic sequence is 27 and the tenth term is 47. What is the value of the first term a and the common difference d ?
- A. $a=2$ and $d=2$
 - B. $a=2$ and $d=5$

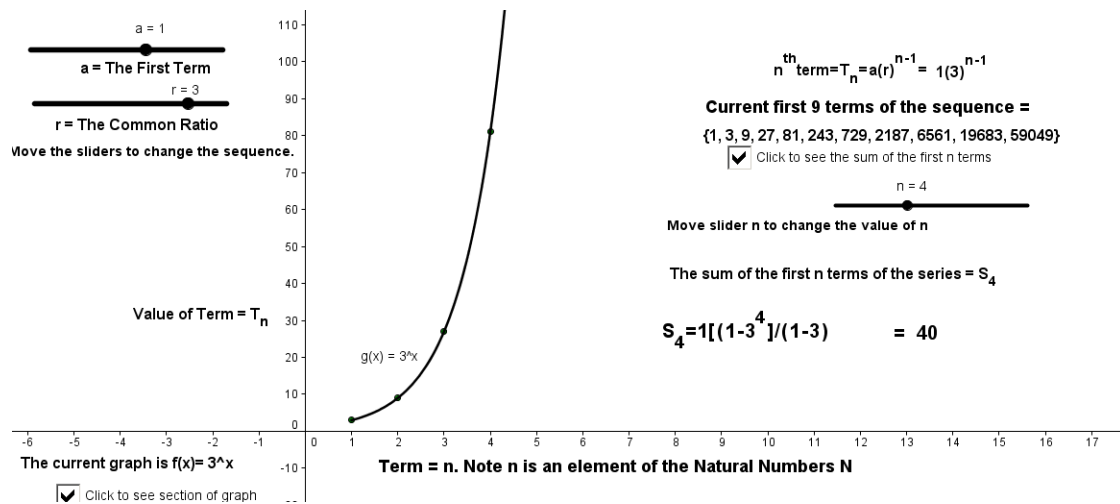
Arithmetic Series Quiz

1. Is an arithmetic series the sum of numbers/terms in the arithmetic sequence?
 - A. No
 - B. Yes
2. Is 1,2,3,4 an arithmetic sequence or series?
 - A. Sequence
 - B. Series
3. Which of the following is an arithmetic series?
 - A. 1,3,6,10,,
 - B. 1,3,5,7,,
 - C. 1+3+6+10
 - D. 1+3+5+7
4. Is the following $\frac{8}{9} + \frac{7}{8} + \frac{6}{7} + \frac{5}{6}$ an arithmetic series?
 - A. Yes
 - B. No
5. Is the following $\frac{8}{9} + \frac{7}{8} + \frac{6}{7} + \frac{5}{6}$ an arithmetic series?
 - A. Yes
 - B. No
6. Find the sum of the first 10 terms of an arithmetic series if $a=2$ and $d=3$.
 - A. 31
 - B. 29
 - C. 155
7. What is the sum of the following series 3+7+11,,,,,,+39?
 - A. 10
 - B. 210
 - C. 253
8. Could the sum of an arithmetic series ever be 0?
 - A. Yes
 - B. No
9. How many terms of the series 2+6+10+... have to be taken so that the sum is 242?
 - A. 10
 - B. 242
 - C. 4
 - D. 11

10. Starting at 3 and taking every 4th natural number after that, find the sum of all such numbers that are less than 100.
- A. 1275
 - B. 99
 - C. 1274
11. A common difference of an arithmetic series can be found by subtracting any term from the previous term.
- A. True
 - B. False
12. Given the first term of an arithmetic sequence is $\log 3$ and the common difference is $\log 3$. What is the sum of the first ten terms.
- A. $\log 3$
 - B. 55
 - C. $55(\log 3)$
13. Kayla got her new mobile phone on the first of April. She sent 1 text that day, 3 texts the next day and 5 texts the next day after that. If this pattern continues how many texts will she send in the month of April that year?
- A. 61
 - B. 59
 - C. 630
 - D. 900
14. Is it true that an alternative method of writing the formula for the sum of the first n Natural numbers is $S_n = \frac{n(n+1)}{2}$?
- A. True
 - B. False
15. Starting with €0, Margo saves €45 per day, how long will it be before she can afford a new (second hand) car costing €3,000?
- A. 66
 - B. 67
 - C. 68
16. Given $S_n = n^2$. What is T_{n+1} ?
- A. $2n-1$
 - B. $2n+1$
 - C. 3

Student Activity: To investigate geometric sequences and series

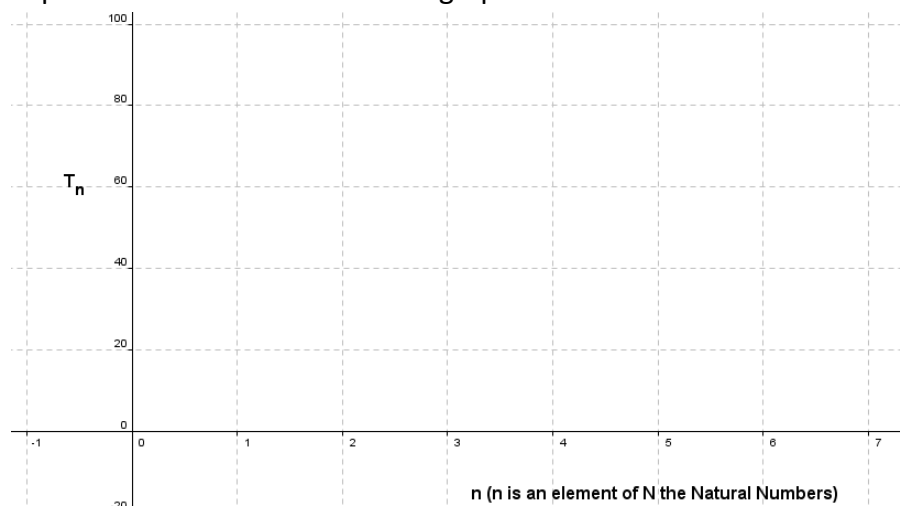
Use in connection with the interactive file, 'Geometric Sequence and Series', on the Student's CD.



- Consider the sequence 1, 2, 4, 8, 16.....
What is the first term of this sequence? How is each term related to the one before it?
What type of sequence is this?

- The first term, "a", of a geometric sequence is equal to 3 and the common ratio, "r", of the sequence is 2.
a. Write down the first 6 terms. Show your calculations.

b. Represent all these terms on the graph below.



- c. Complete the following table using the geometric sequence in the graph and from your data and the table above determine if the pattern formed is linear, quadratic, exponential or none of these.

n	n^{th} term	Change	Change of the change	Ratio of $(n+1)^{\text{th}}$ term to the previous term
1				
2				
3				
4				
5				

The pattern formed is _____

3. The first term of a geometric sequence is equal to a and the common ratio of the sequence is r .

a. Write down the first 6 terms in terms of a and r .

b. From the pattern of the terms in a above, determine a formula for T_n , the n th term of a geometric sequence.

4. What is the ratio between T_n and T_{n+1} of any geometric sequence where T_n is its n th term and T_{n+1} is its $(n+1)$ th term?

5. When, if ever, is T_n less than T_{n+1} , in a geometric sequence?

6. What is the difference between a geometric sequence and geometric series?

7. Using the formula $S_n = a \left(\frac{1-r^n}{1-r} \right)$, find the sum of the first 10 terms of a geometric series given $a=2$ and $r=3$. Check this using your calculator.

8. Given that a geometric series is $a+ar+ar^2+ar^3+\dots+ar^{n-1}$, prove that S_n , the sum of the first n terms of a geometric series, is equal to $S_n = a \left(\frac{1-r^n}{1-r} \right)$. If necessary check the file 'Proof of the Sum of Geometric Series'.

9. If you toss a coin once you get 2 possible outcomes, toss it twice you get 4 possible outcomes and toss it three times you get 8 possible outcomes. If this pattern continues, how many outcomes would one get for 20 tosses? Show your calculations.

10. An antique costing €4,000 appreciates each year by 6% of the value it was at the beginning of that year. Find its value after 11 years. Show your calculations.

11. The birth rate in an area is increasing by 6% per annum. If 20,000 children were born in this area in 2009

a) How many children will be born in 2018 in this area?

b) How many children in total will be born in this area between the years 2009 to 2018 inclusive?

Geometric Sequences Quiz

1. Given this pattern continues, is the sequence 2, 4, 8, 16, ..., a geometric sequence?
 - A. Yes
 - B. No
2. Given 2, 8, 32, ... is a geometric sequence could 231 be a term in this sequence?
 - A. Yes
 - B. No
3. Given the first term of a geometric sequence is 23 and the common ratio is 2, what is the 3rd term?
 - A. 46
 - B. 92
 - C. 69
4. What are the next 3 terms of the geometric sequence 729, 243 and 81, assuming the pattern continues?
 - A. 162, 0, -162
 - B. 78, 75, 72
 - C. 9, 3, 1
 - D. 27, 9, 3
5. Calculate the common ratio r of this sequence if the first term is 5 and the 8th term is 640.
 - A. 40
 - B. 2
 - C. 64
6. The seventh term of a geometric sequence is 1458 and the fourth term is 54. What are the possible values for a and r ?
 - A. $a = 1$ and $r = 3$
 - B. $a = 2$ and $r = 3$
 - C. $a = -2$ and $r = -3$
7. Given the 6th term of a geometric sequence is 486, which of the following is false?
 - A. $a = -2$ $r = -3$
 - B. $a = 2$ $r = -3$
 - C. $a = 2$ $r = 3$

8. For all geometric sequences is the $n+1$ th term divided by the n th term always a constant?
 - A. Yes
 - B. No
9. Is the Fibonacci sequence $\{1, 1, 2, 3, 5, 8, 13, 21, \dots\}$ a geometric sequence?
 - A. Yes
 - B. No
10. What is the first term of the geometric sequence 2, 4, 8, 16 that is greater than 200 equal to?
 - A. 202
 - B. 256
 - C. 201
 - D. 512
11. If one continually folds a square piece of paper of area 64 cm squared in half, does the list of the visible areas form a geometric sequence?
 - A. No
 - B. Yes
12. Given a is the first term of a geometric sequence and 2 is the common ratio, what is the 5th term in terms of a ?
 - A. $32a$
 - B. $16a$
 - C. $5a$
 - D. $4a$
13. The 5th term of a geometric sequence is 243 and the 6th term of the same sequence is 729, what is the value of the second term of this sequence?
 - A. 3
 - B. 2
 - C. 9
14. Given " r " the common ratio is a positive integer, does each consecutive term in the geometric sequence get smaller or larger?
 - A. Smaller
 - B. Larger.
15. Given " r " the common ratio is a positive fraction less than 1 and the first term is a positive integer does each consecutive term in the geometric sequence get smaller or larger?
 - A. Smaller
 - B. Larger

16. Given " r " the common ratio is negative then each term in the geometric sequence is negative.
 - A. True
 - B. False
17. Given " a " the first term is negative and " r " the common ratio is positive then all the terms in the geometric sequence will be negative.
 - A. True
 - B. False
18. Given an investment of €100 in a bank earns interest at the rate of 10% each year does the sequence generated by the value of the investment after n years form a geometric sequence.
 - A. True
 - B. False
19. Given the number of bacteria in a Petri dish is 10 and they double themselves every minute, does the number of bacteria in the Petri dish after n minutes form a geometric sequence?
 - A. No
 - B. Yes
20. Given the number of bacteria in a Petri dish is 10 and they double themselves every minute, how many bacteria will be in dish after 6 minutes?
 - A. 32000
 - B. 60
 - C. 320
 - D. 640

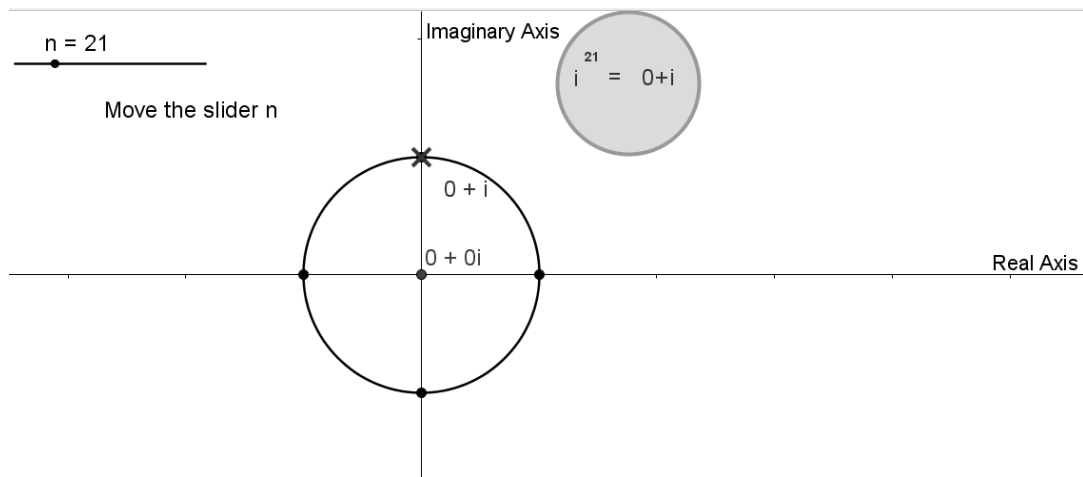
Geometric Series Quiz

1. The sum of the terms in a sequence is known as a Series.
 - A. True
 - B. False
2. What is the formula for the sum of the first n terms of a geometric sequence?
 - A. ar^{n-1}
 - B. ar^n
 - C. $\frac{a(1-r^n)}{1-r}$
3. Which letter in the formula for the sum of the first n terms of a geometric series denotes the first term of the geometric series?
 - A. a
 - B. n
 - C. r
4. What is the value of r the common ratio of this series $1+5+25+125,,?$
 - A. 1
 - B. 4
 - C. 5
 - D. 156
5. Is the following a geometric series $1, 2, 4, 8, 16,,,?$
 - A. No
 - B. Yes
6. The sum of the terms in a sequence is known as a Series.
 - A. True
 - B. False
7. Find the sum of the first 4 terms of the geometric series $1 +.5 +.25+$
 - A. 1.875
 - B. .125
 - C. 1.75

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Student Activity: To investigate i^n

Use in connection with the interactive file, 'i to power of n', on the Student's CD.



1. Using the interactive file calculate each of the following:

		Answer
a.	i^2	
b.	i^3	
c.	i^4	
d.	i^5	
e.	i^6	
f.	i^7	
g.	i^8	
h.	i^{20}	
i.	i^{21}	
j.	i^{22}	
k.	i^{23}	

l.	i^{200}	
m.	i^{201}	
n.	$i^4 \times i^2$	
o.	i^4 / i^2	
p.	i^0	
q.	$i^4 \times i^0$	

2. Multiplying by i causes a rotation of how many degrees?

3. Multiplying by i^2 causes a rotation of how many degrees?

4. Multiplying by i^3 causes a rotation of how many degrees?

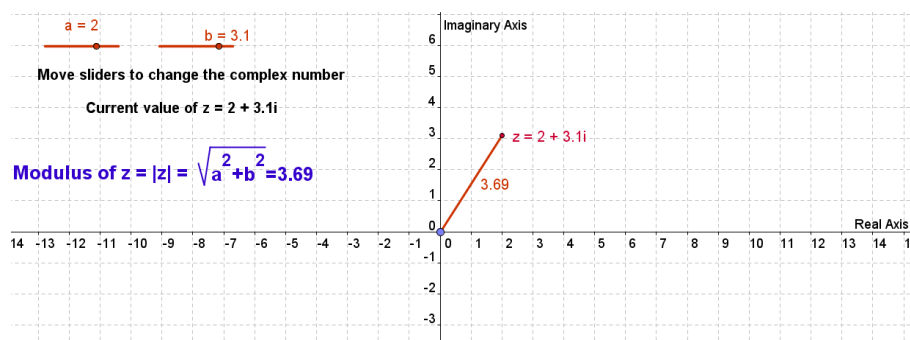
5. Multiplying by i^4 causes a rotation of how many degrees?

6. Based on your previous answers outline a general rule for calculating i^n ?

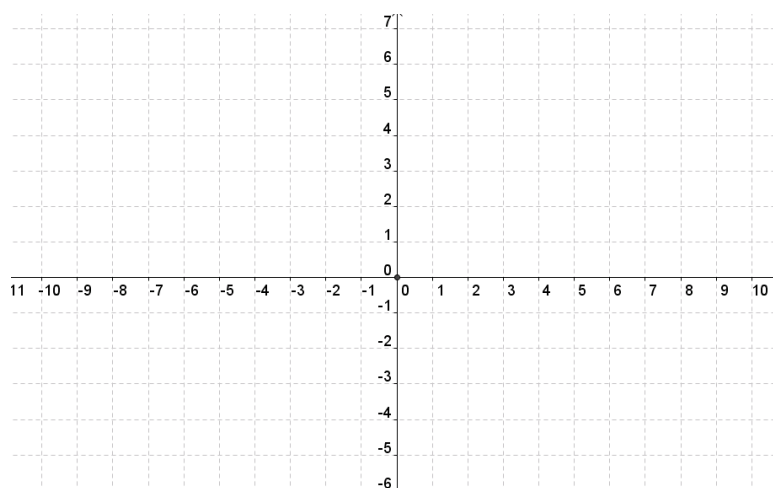
7. For all values of n , what is the distance of i^n from the origin?

Student Activity: To investigate the Argand diagram and modulus of complex numbers

Use in connection with the interactive file, 'Argand diagram and modulus of a complex number', on the Students CD.



- Place the following numbers on the Argand diagram and calculate the modulus in each case.



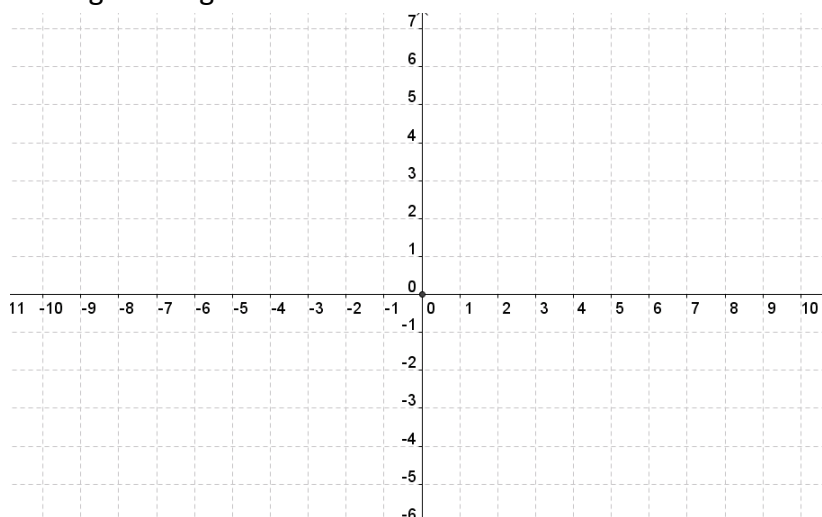
		Modulus = $ z $
a)	$z_1 = 3 + 2i$	
b)	$z_1 = 2 + 3i$	
c)	$z_1 = -3 + 4i$	
d)	$z_1 = -2 - 4i$	
e)	$z_1 = i$	
f)	$z_1 = -2 - i$	
g)	$z_1 = 1 + \sqrt{25}i$	

2. Write in your own words, what is meant by the modulus of a complex number.

3. What is the mathematical notation for the modulus of a complex number z ?

4. List four complex numbers with a modulus equal to 4.

5. Is $|z_1| + |z_2|$ equal to $|z_1 + z_2|$? Always? Sometimes? Never? Explain using examples plotted on the Argand diagram.



6. What do you notice about $|a+bi|$ and $|a-bi|$?

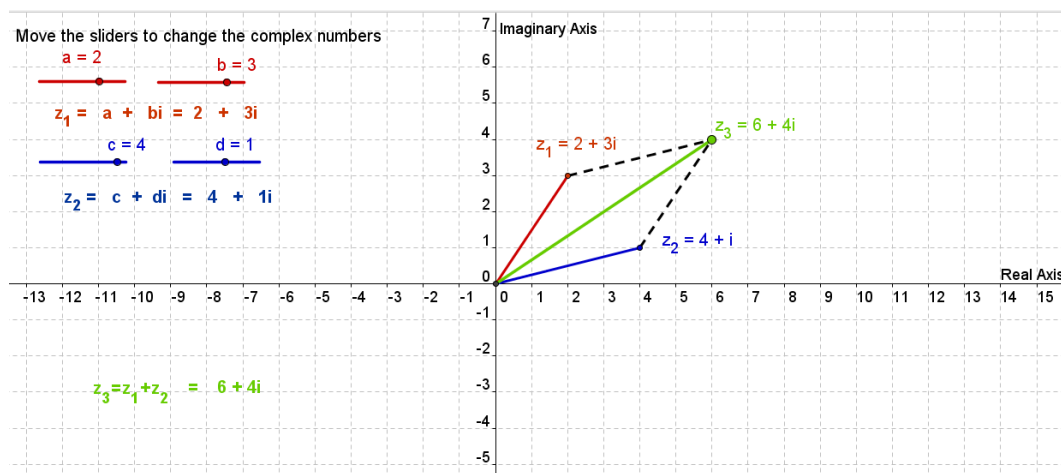
7. Describe how you could show all the complex numbers that have a modulus of 3.

8. What do all numbers on the Real Axis of the Argand diagram have in common?

9. What do all numbers on the Imaginary Axis of the Argand diagram have in common?

Student Activity: To investigate the addition of complex numbers

Use in connection with the interactive files, 'Addition of complex numbers', and 'Addition by translation of a complex number', on the Student's CD.



1. Add the following complex numbers and check your answers using the interactive file "Addition of Complex numbers".

		Calculate $z_1 + z_2$
a)	$z_1 = 3 + 2i$ and $z_2 = 1 + 4i$	
b)	$z_1 = 2 + 3i$ and $z_2 = 1 + 3i$	
c)	$z_1 = 2 + 4i$ and $z_2 = 1 - 3i$	
d)	$z_1 = 2 + 4i$ and $z_2 = -1 - 2i$	
e)	$z_1 = -3 + 4i$ and $z_2 = 1 - 2i$	
f)	$z_1 = -2 - 4i$ and $z_2 = 1 - 3i$	
g)	$z_1 = -1 - 3i$ and $z_2 = -2 - 1i$	
h)	$z_1 = i$ and $z_2 = 2 + i$	

i)	$z_1 = i$ and $z_2 = i$	
j)	$z_1 = i$ and $z_2 = -i$	
k)	$z_1 = 1$ and $z_2 = -1$	
l)	$z_1 = 1$ and $z_2 = -i$	
m)	$z_1 = -1 - i$ and $z_2 = -2 - i$	
n)	$z_1 = -1 - i$ and $z_2 = -2 - i$	
o)	$z_1 = 1 + i$, $z_2 = -2 - 2i$ and $z_3 = 2 + 3i$	
p)	$z_1 = 1 + \sqrt{25}i$ and $z_2 = -2 - \sqrt{36}i$	

2. What shape is formed when you add two complex numbers?

3. What complex number would you need to add to $2 + 3i$ to get $0 + 0i$?

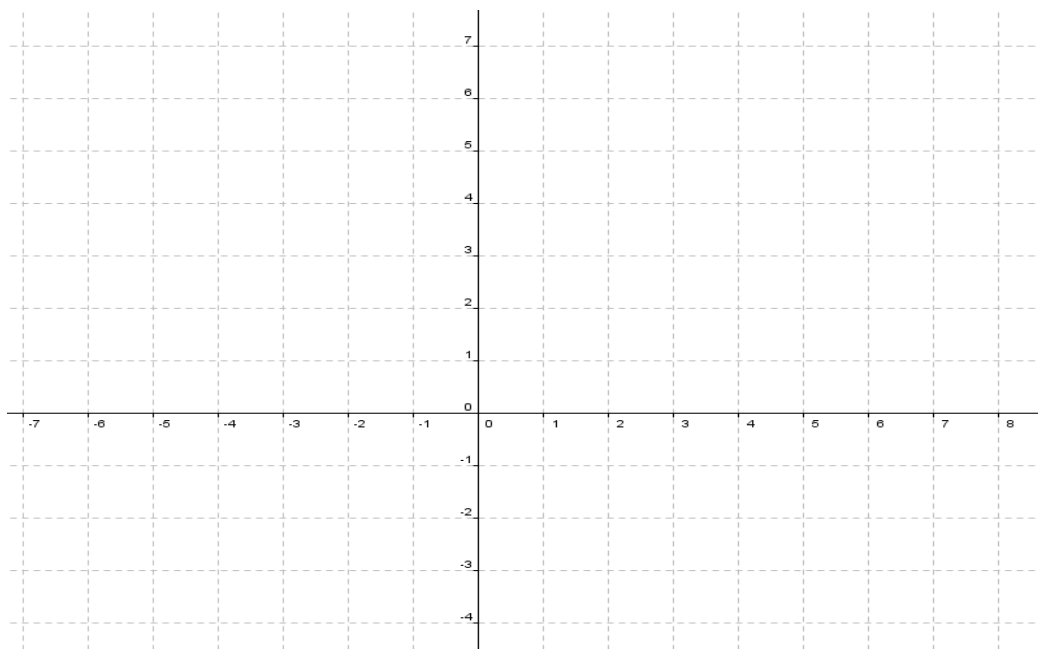
4. If two complex numbers z_1 and z_2 are added together to give $4 + 6i$, list four values z_1 and z_2 could have.

5. Is the addition of complex numbers associative? Explain your answer.

6.

a. Plot the following complex numbers in the Argand Diagram.

- i. $2 + 4i$
- ii. $2 - 2i$
- iii. $-3 + i$
- iv. $-2 - 3i$



b. Add $2+1i$ to each of the complex numbers in section a. of this question (It is not necessary to show the parallelograms).

i. _____

ii. _____

c. Draw a directed line (a line with an arrow indicating direction) between each complex number and its corresponding number with $2+1i$ added to it. What do you notice?

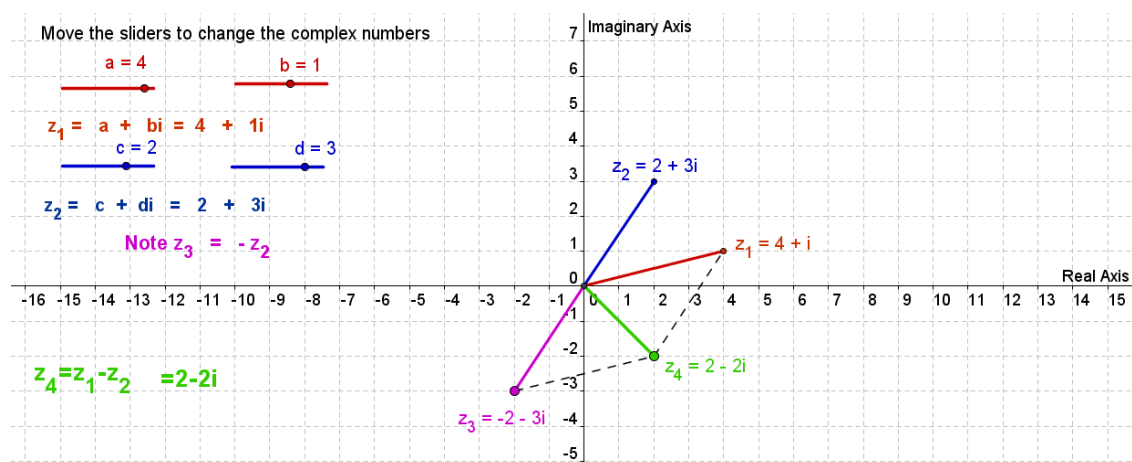
i. _____

ii. _____

d. What would have happened if instead of adding the complex number $2+1i$ to each of the complex numbers above you had subtracted $2+1i$?

Student Activity: To investigate subtraction of complex numbers

Use in connection with the interactive file, 'Subtraction of complex numbers', on the Student's CD.



1. Calculate $z_1 - z_2$ in each of the following cases and check your answer using the interactive file 'Subtraction of Complex numbers'.

		$z_1 - z_2$
a)	$z_1 = 3 + 2i$ and $z_2 = 1 + 4i$	
b)	$z_1 = 2 + 4i$ and $z_2 = 1 - 3i$	
c)	$z_1 = 2 + 4i$ and $z_2 = -1 - 2i$	
d)	$z_1 = -3 + 4i$ and $z_2 = 1 - 2i$	
e)	$z_1 = -1 - 3i$ and $z_2 = -2 - 1i$	
f)	$z_1 = i$ and $z_2 = 2 + i$	
g)	$z_1 = i$ and $z_2 = i$	
h)	$z_1 = i$ and $z_2 = -i$	
i)	$z_1 = 1$ and $z_2 = -1$	
j)	$z_1 = 1$ and $z_2 = -i$	
k)	$z_1 = -1 - i$ and $z_2 = -2 - i$	

l)	$z_1 = -1 - i$ and $z_2 = -2 - i$	
m)	$z_1 = 1 + i$ and $z_2 = -2 - 2i$	
n)	$z_1 = 1 + \sqrt{25}i$ and $z_2 = -2\sqrt{9}i$	

2. If the complex number $z = a + bi$, what is $-z$?

3. How does the complex number $-z$ differ from the complex number z ?

4. What complex number gives $0 + 0i$ when subtracted from $2 + 3i$?

5.

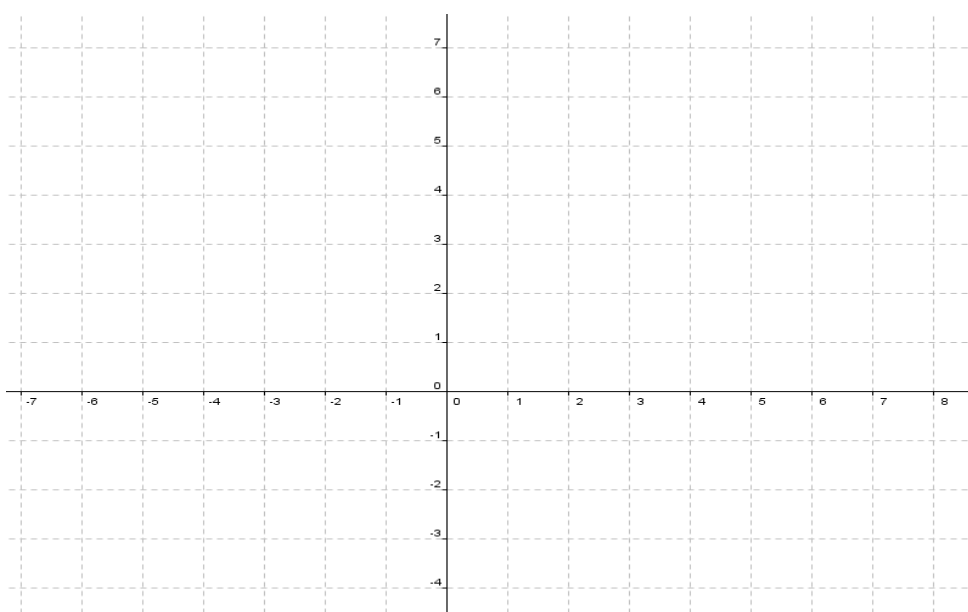
a. Plot the following complex numbers in the Argand Diagram.

i. $2 + 2i$

ii. $2 - 2i$

iii. $3 + 2i$

iv. $-2 + 3i$



- b. Subtract $2+3i$ from each of the complex numbers in section a. of this question.

- c. Draw a directed line (a line with an arrow indicating direction) between each complex number and its corresponding number with $2+3i$ subtracted from it. What do you notice?

6. Is $z_1 - z_2$ always the same as $z_2 - z_1$? Explain your answer.

Student Activity: To investigate the conjugate of a complex number

Use in connection with the interactive file, 'Conjugate of a complex number', on the Student's CD.



N.B. Note the complex conjugate of a complex number z is written as $\bar{z}$

- Find the complex conjugate of the following numbers and check your answers using the interactive file.

		Calculate $\bar{z}$.
a.	$z_1 = 3 + 2i$	
b.	$z_1 = 2 + 3i$	
c.	$z_1 = 1 - 3i$	
d.	$z_1 = -1 - 2i$	
e.	$z_1 = -3 + 4i$	
f.	$z_1 = -i$	
g.	$z_1 = 4$	
h.	$z_1 = -1 - i$	
i.	$z_1 = 1 + \sqrt{25}i$ and $z_2 = -2 - \sqrt{36}i$	

2. On the Argand diagram, which transformation gives the conjugate of a complex number?

3. If $2-4i$ is the complex conjugate of Z_1 , what is Z_1 ?

4. Does every complex number have a unique complex conjugate? Explain.

5. Is the modulus of a complex number equal to the modulus of its complex conjugate? Explain your answer.

6. What type of number do you always get when you add a complex number to its conjugate?

7. What can you say about two complex numbers if the sum of these two complex numbers is real?

8. What type of number do you always get when you subtract a number's complex conjugate from that number?

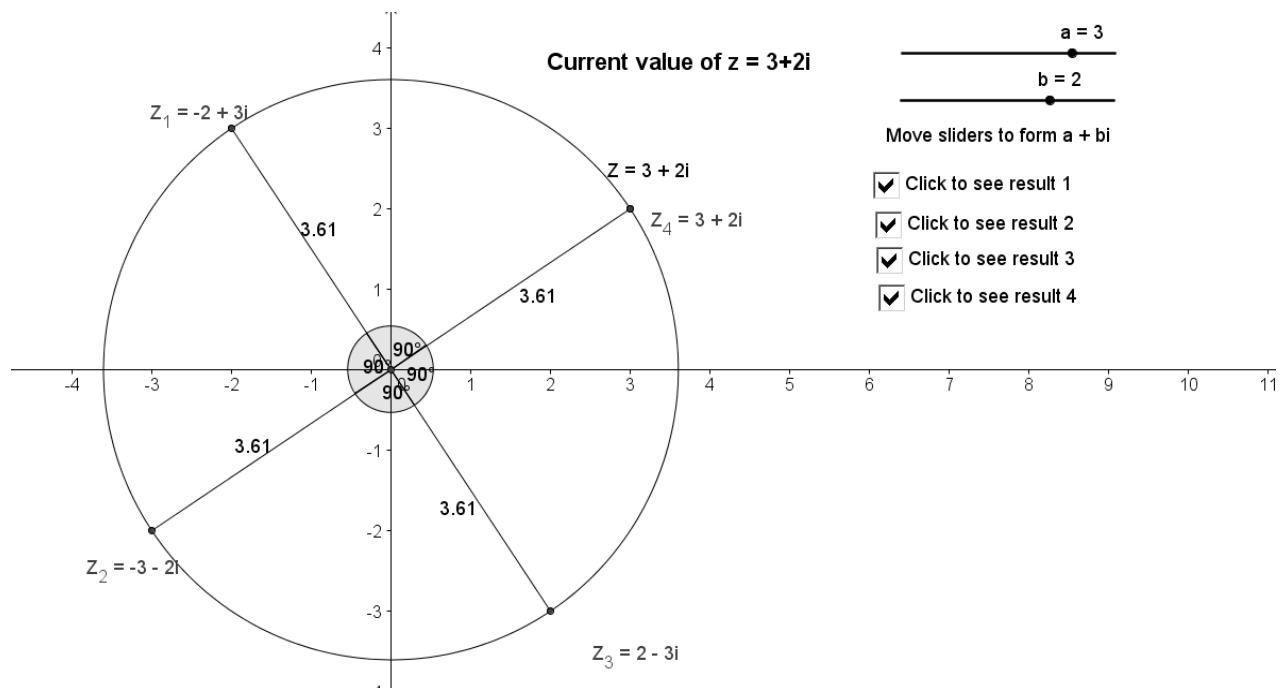
9. Multiply $2+3i$ by its complex conjugate. What do you notice?

10.
 - a. Multiply $a + bi$ by its complex conjugate. What do you notice?

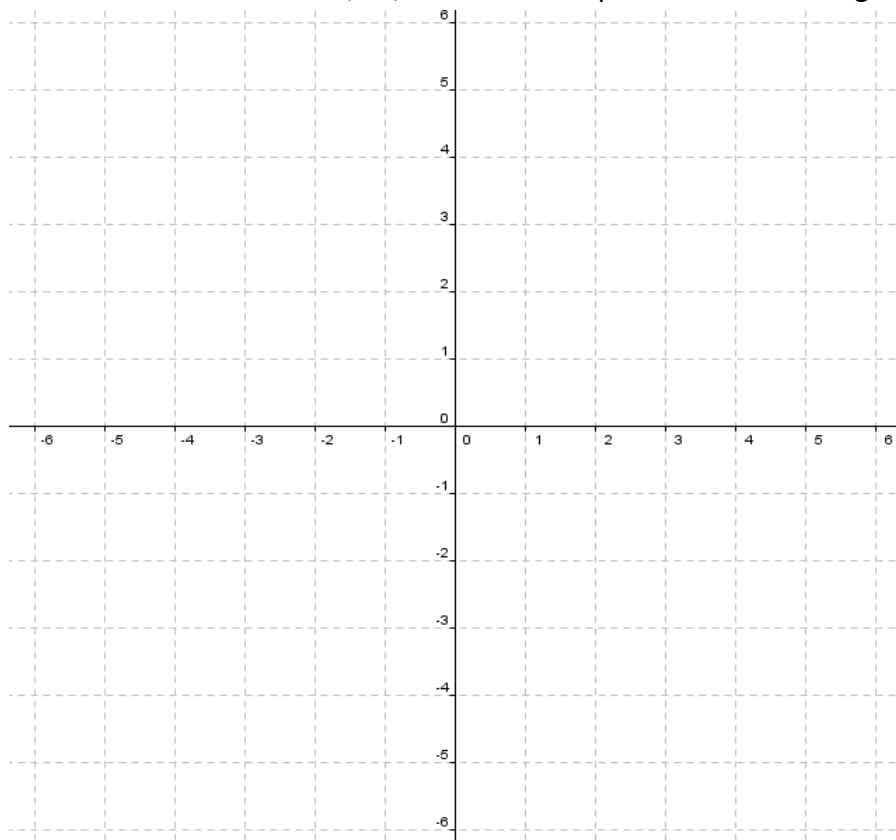
 - b. Is this always the case when you multiply a complex number by its complex conjugate?

Student Activity: To investigate multiplication by i

Use in connection with the interactive files, 'Multiplication by i ', on the Student's CD.

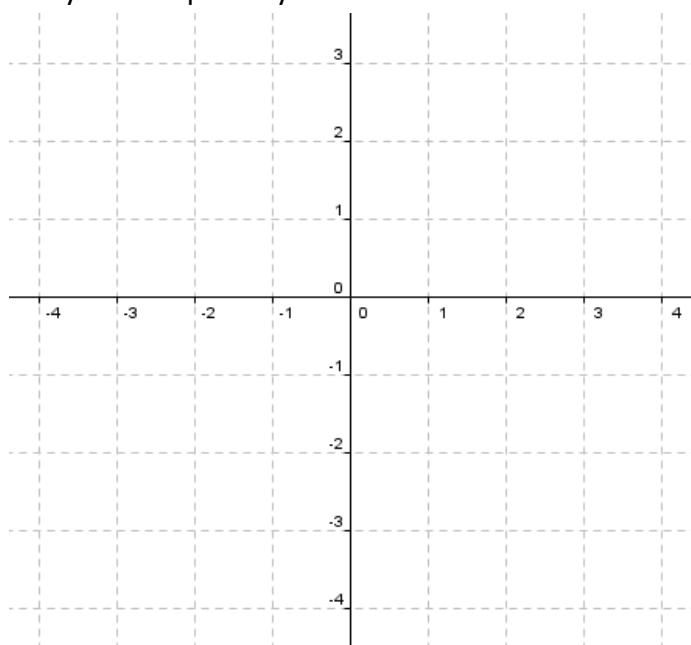


- Given $z = 3 + 2i$. Calculate iz , i^2z , i^3z and i^4z and plot them on the Argand diagram.



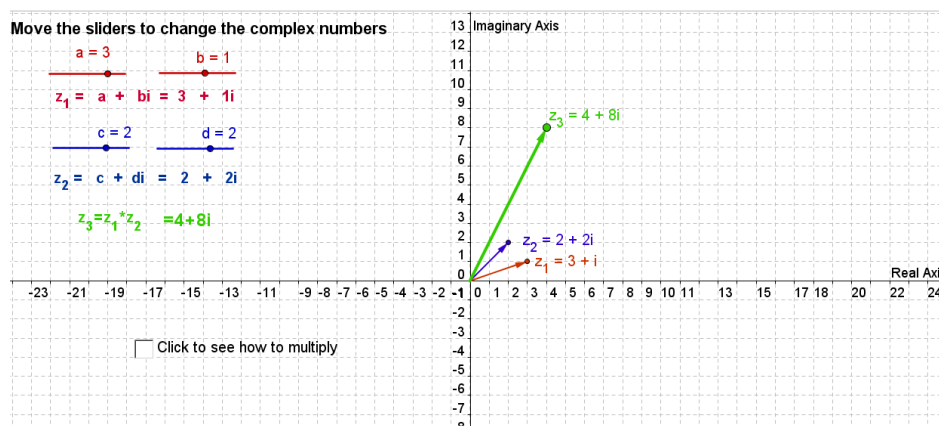
- Investigate what happened geometrically when you multiplied z by i to get iz .

- b. Investigate what happened geometrically when you multiplied iz by i to get i^2z .
-
- c. Investigate what happened geometrically when you multiplied i^2z by i to get i^3z .
-
- d. Investigate what happened geometrically when you multiplied i^3z by i to get i^4z . How does this relate to z ?
-
2. With the help of the interactive file, explain what happens geometrically when you multiply any complex number by i .
-
3. With the help of the interactive file, explain what happens geometrically when you multiply any complex number by i^2 .
-
4. With the help of the interactive file, explain what happens geometrically when you multiply any complex number by i^3 .
-
5. With the help of the interactive file, explain what happens geometrically when you multiply any complex number by i^4 .
-
6. Multiply $2+3i$ by $-i$ and represent on the Argand diagram. Explain what happened geometrically when you multiplied by $-i$.



Student Activity: To investigate the multiplication of complex numbers

Use in connection with the interactive files, 'Multiplication of complex numbers' and 'Multiplication of complex numbers 2', on the Students CD.



1. Calculate Z_1 multiplied by Z_2 in each of the following cases and check your answer using the interactive file "Multiplication of complex numbers 1".

		$Z_1 * Z_2$
a.	$z_1 = 3 + 2i$ and $z_2 = 1 + 4i$	
b.	$z_1 = 2 + 4i$ and $z_2 = 1 - 3i$	
c.	$z_1 = -3 + 4i$ and $z_2 = 1 - 2i$	
d.	$z_1 = -1 - 3i$ and $z_2 = -2 - 1i$	
e.	$z_1 = i$ and $z_2 = 2 + i$	

f.	$z_1 = i$ and $z_2 = i$	
g.	$z_1 = i$ and $z_2 = -i$	
h.	$z_1 = 1$ and $z_2 = -1$	
i.	$z_1 = 1$ and $z_2 = -i$	
j.	$z_1 = 1 + \sqrt{25}i$ and $z_2 = -2\sqrt{9}i$	
k.	$Z_1 = -1 - i$ and $Z_2 = -2 - i$	
l.	$Z_1 = -2 - i$ and $Z_2 = -1 - i$	

2. Given $z_1 = a + ib$ and $z_2 = c + id$ investigate if $z_1 z_2 = z_2 z_1$ for all complex numbers.

Use the interactive file **Multiplication of complex numbers 2** to answer the following questions.

3. When you multiply a complex number by a real number what is the relationship between the angle that the original complex number made with the Real Axis and the angle the answer makes with the Real Axis?

4. When you multiply a complex number by a real number what is the relationship between the modulus of the original complex number and the modulus of the product?

5. When you multiply two complex numbers, what is the relationship between the angles that each of these complex numbers makes with the Real Axis and the angle that their product makes with the Real Axis?

6. When you multiply two complex numbers, what is the relationship between the modulus of each of these complex numbers and the modulus of their product?

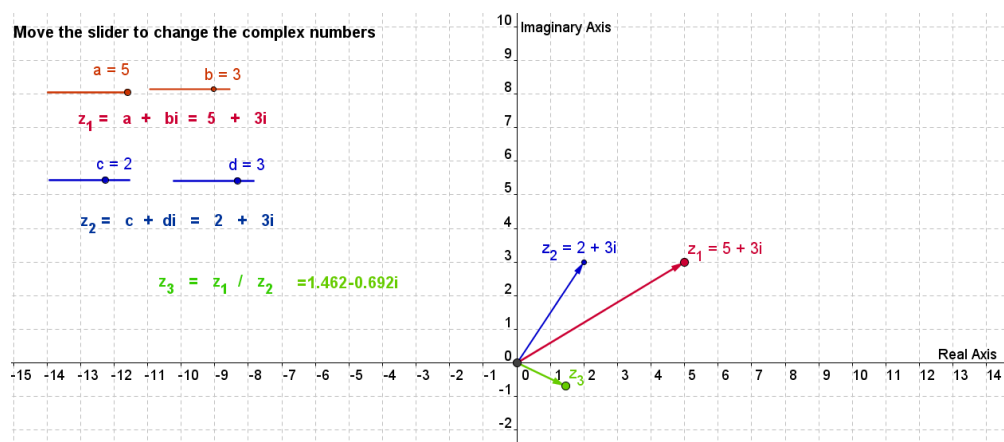
7. When you multiply a complex number by its complex conjugate, what angle does the solution make with the positive Real Axis?

8. By how many degrees is the number $a + bi$ rotated when it is multiplied by i ?

9. When you multiply any complex number $a + bi$ by i how does the modulus of the initial number relate to the modulus of the product?

Student Activity: To investigate division of complex numbers

Use in connection with the interactive file, 'Division of complex numbers' and 'Division of complex numbers2', on the Student's CD.



- Calculate z_1 divided by z_2 in each of the following cases and check your answer using the interactive file.

		$\frac{z_1}{z_2}$
a.	$z_1 = 2$ and $z_2 = 1 + 4i$	
b.	$z_1 = 2 + 4i$ and $z_2 = 1 - 3i$	
c.	$z_1 = -3 + 4i$ and $z_2 = 1 - 2i$	
d.	$z_1 = i$ and $z_2 = 2 + i$	

e.	$z_1 = i$ and $z_2 = i$	
f.	$z_1 = -1 - i$ and $z_2 = -2 - i$	
g.	$z_1 = 1 + \sqrt{25}i$ and $z_2 = -2 + \sqrt{9}i$	

2. Given two complex numbers explain how you calculate $\frac{z_1}{z_2}$.

3. If $z = a + bi$, calculate $\frac{z}{\bar{z}}$ where $\bar{z}$ is the complex conjugate of z .

4. If $z_1 = -1 - i$ and $z_2 = -2 - i$ investigate whether $\frac{z_2}{z_1} = \frac{z_1}{z_2}$?

5. When dividing complex numbers, what do you multiply the numerator and denominator by? Explain why you do this.

6. If $(3 + 4i)(x + yi) = 5 + 2i$, find x and y .

7. If $(2 - 3i)(x + yi) = 5 + 4i$, find x and y .

8. Calculate $\left| \frac{2 + 3i}{i} \right|$.

9. Calculate $\left| \frac{1 + i}{1 - i} \right|$.

10. Using the interactive file investigate whether, when one complex number is divided by another, the difference between the angles they make with the Real Axis is equal to the angle the quotient makes with the Real Axis. Do you think that this is true in general?

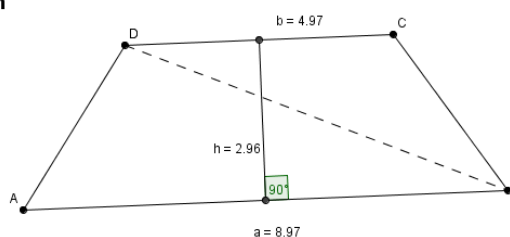
Student Activity: To investigate trapeziums

Use in connection with the Interactive file, 'Trapezium', on the Student's CD.

Move point A to change the trapezium

☒ Click to see what is a trapezium

A trapezium is a quadrilateral with a set of parallel lines.



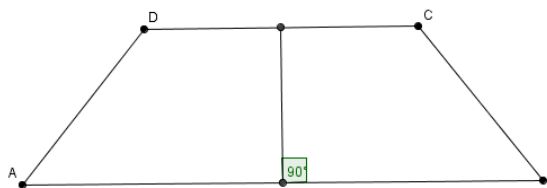
☒ Click to see area of current trapezium

The area of the current trapezium is = 20.64

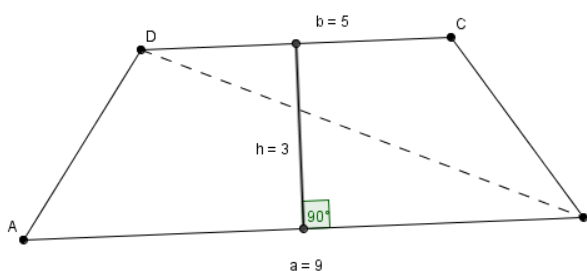
Correct to 2 decimal places

1. What is meant by a trapezium? (See interactive file if necessary.)

2. Which sides of the trapezium below are parallel?



- 3.

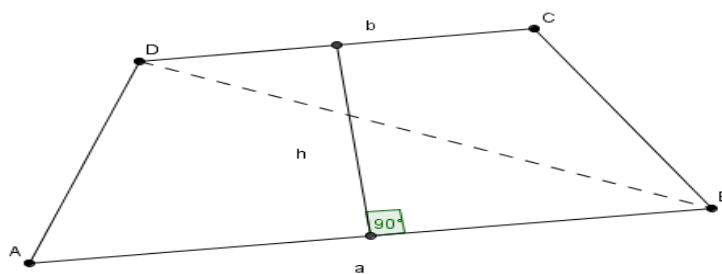


- a. What is the perpendicular height of the triangle ABD? _____
- b. What is the area of the triangle ABD? Show your calculations.

- c. What is the perpendicular height of the triangle BCD? _____
- d. What is the area of the triangle BCD? Show your calculations.

- e. Hence, what is the area of trapezium ABCD? Show your calculations.

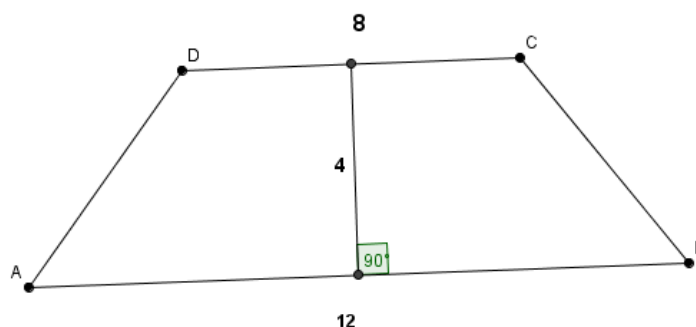
4.



- a. Find the area of the trapezium $ABCD$ in terms of a , b and h . Show your calculations.

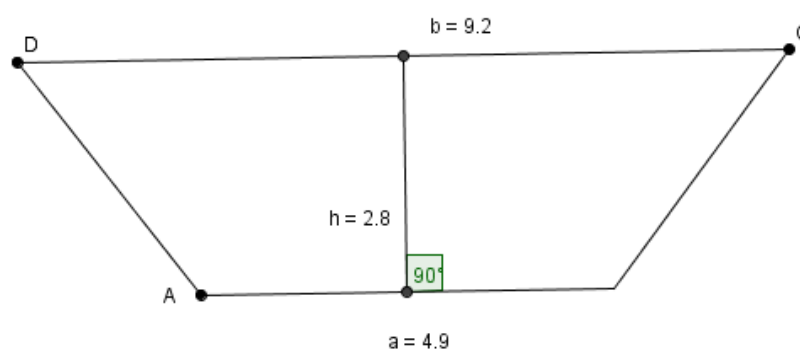
- b. Is the formula you got in a above true for all trapeziums?

5.



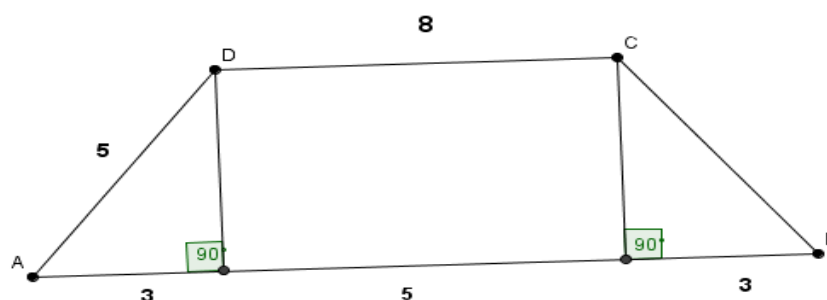
Find the area of the trapezium $ABCD$ represented in the above diagram. Show your calculations.

6.



Find the area of the trapezium $ABCD$ represented in the above diagram. Show your calculations.

7.



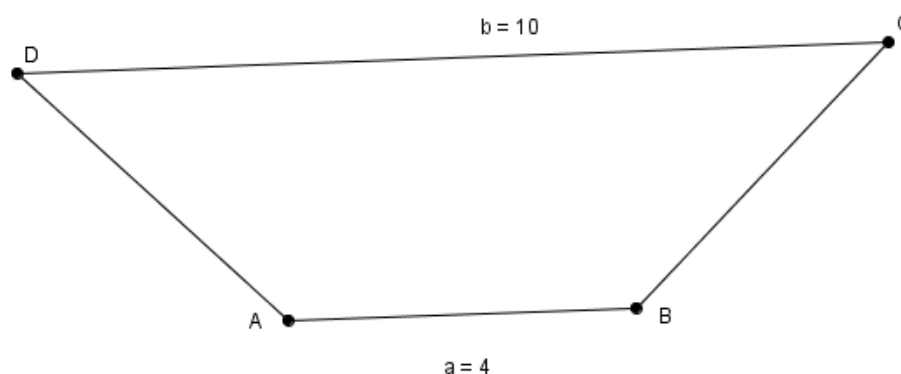
- a. Find the area of the trapezium represented in the above diagram. Show your calculations.

- b. Find the perimeter of the above trapezium. Show your calculations.

8. A trapezium of the following shape is to be cut from a piece of material. What is the area of material that will be required and how much material will be wasted?



9. Find the perpendicular height of the following trapezium given the area is 20.8 cm^2 .

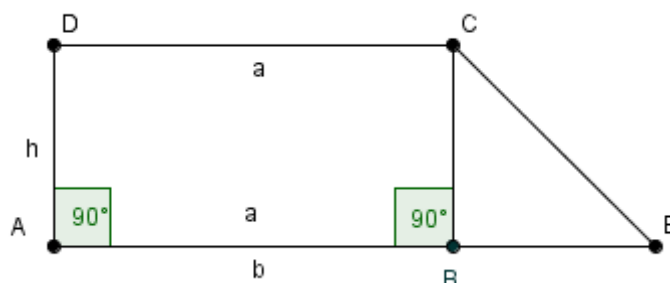


10. If you found a rubbish skip, with no lid, whose sides are in the shapes of trapeziums, what lengths would you need to measure to get the surface area of the skip?

Student Activity: To investigate the Trapezoidal Rule

Use in connection with the interactive file, 'Trapezoidal Rule Semi Circle' on the student's CD.

1.



Note a is the length of AB and b is the length of AE.

- a. Given that $|AB| = |DC|$ and that AE is a line segment, name the two geometric shapes which together make up the figure AECD.

- b. What is the area of ABCD in terms of a and h ?

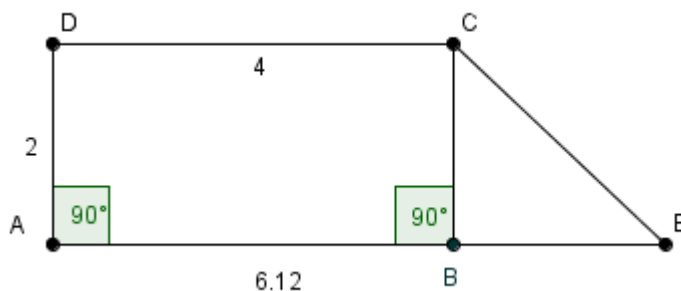
- c. What is the area of BCE in terms of a , b and h ?

- d. What special name is given to the quadrilateral AECD? If you don't know, see page 8 of the *formulae and tables* booklet.

- e. What is the area of AECD? Show your calculations.

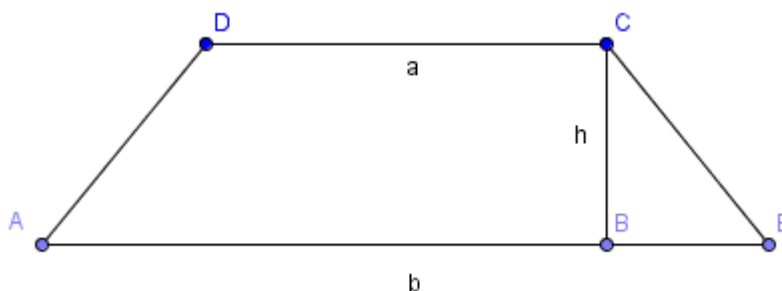
- f. Look at your *formulae and tables booklet* page 8 and check the formula there. Does your answer in part e agree with it?

- g. Find the area of this trapezium.



Note: $|AE| = 6.12$

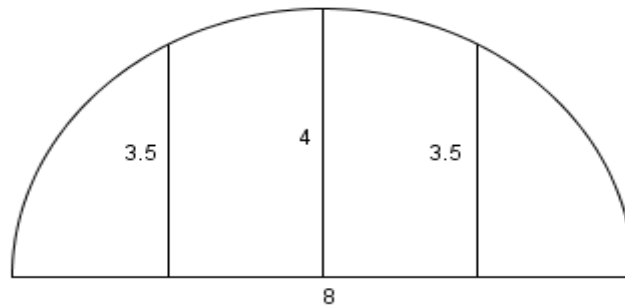
- h. Is the figure AECD a trapezium? Explain your answer.



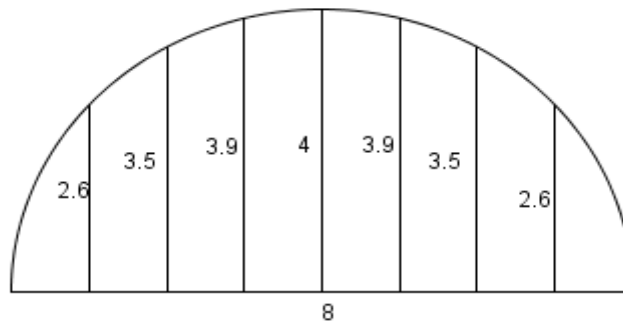
Note: b is the length of AE and BC is perpendicular to AE.

- i. What is the area of the above shape in terms of a , b and h .

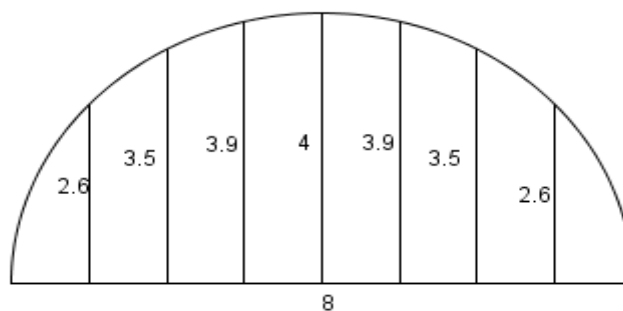
2. Using the fact that the vertical lines in the semi circle are equally spaced, find the approximate area of this semi circle using triangles and trapeziums.



3. Using the fact that the vertical lines in the semi circle are equally spaced, find the approximate area of this semi circle using triangles and trapeziums.

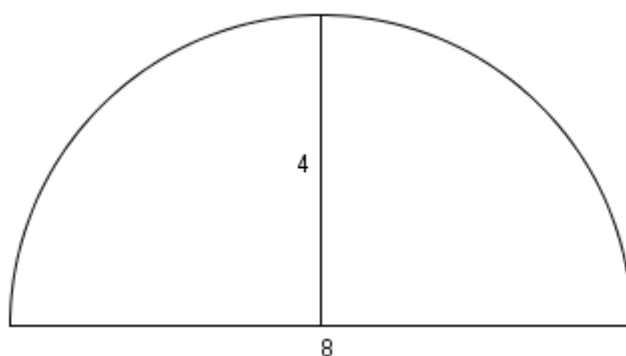


4. Use the Trapezoidal Rule in your *formulae and tables* to find the area of this semi circle below.

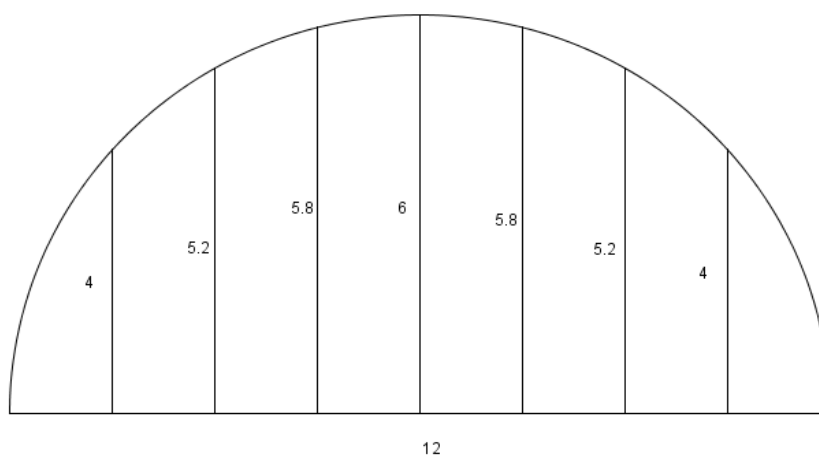


5. Without using the formula for the area of a semi circle, how would you get a more accurate answer than the answer you got in the previous question?

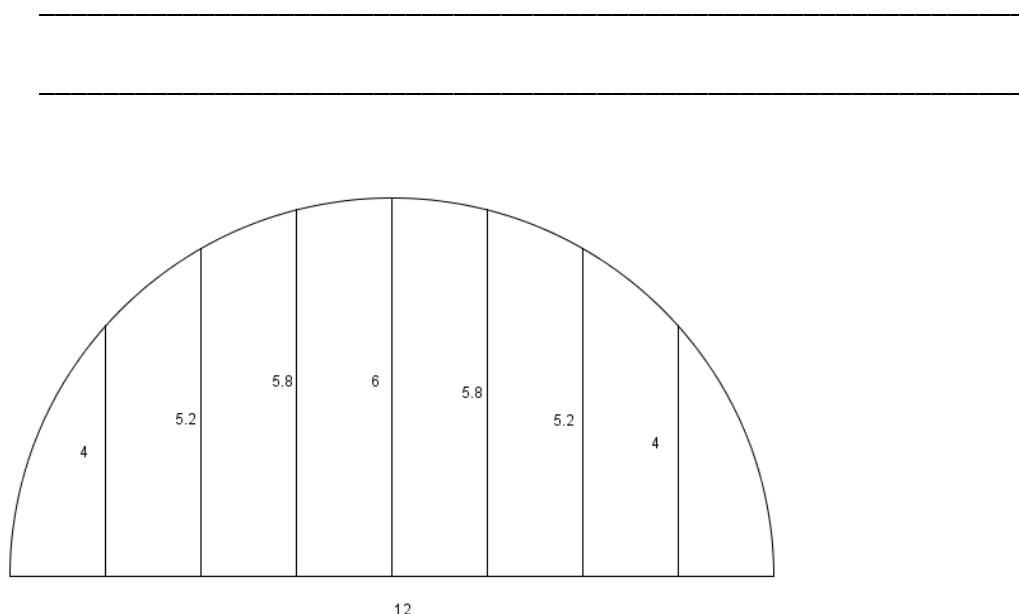
6. Use 3.14 as π to find the area of the semi circle below. Now compare your answer to the answers you got for Q.4 and Q.5.



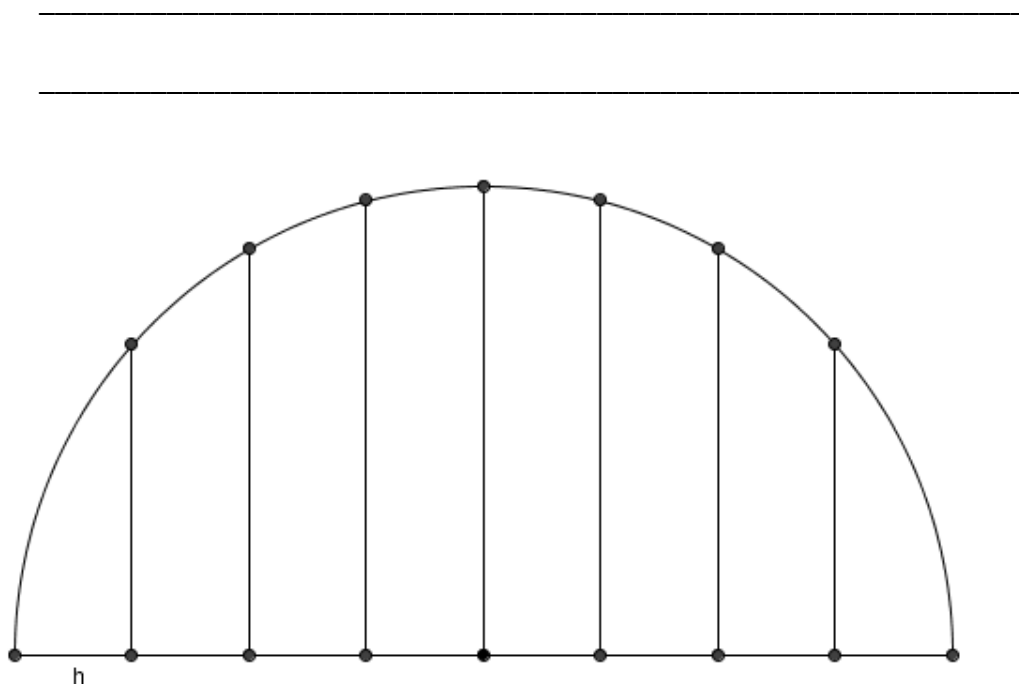
7. Using the fact that the vertical lines in the semi circle are equally spaced, find the approximate area of this semi circle using triangles and trapeziums.



8. Use the formula for the Trapezoidal Rule found in your *formulae and tables* booklet to find the area of the semi circle below.



9. By measuring with your ruler, find the approximate area of the semicircle below by estimating it with trapeziums and triangles. Use the table below and use a and b as the lengths of the parallel sides in the trapeziums. (The triangles at each end can be treated as trapeziums with one parallel side of zero length.) Shade in the area you are “missing” when you do this.



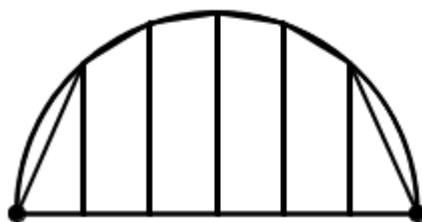
Shape	a	b	h	$A = h\left(\frac{a+b}{2}\right)$
1	0			
2				
3				
4				
5				
6				
7				
8				
Total Area of all the trapeziums and triangles				

- Use the formula** : Area of a semicircle $A = \frac{\pi r^2}{2} =$ _____
- What is the error in using the approximation with the trapeziums and triangles:

- What is the % error in using this approximation? _____
- How could the approximation be improved? _____

Higher Level

10. Deriving the formula for the Trapezoidal Rule (for 6 intervals)



Instead of giving measurements for the parallel sides of the trapezium, label them as y_1 , y_2 , etc.

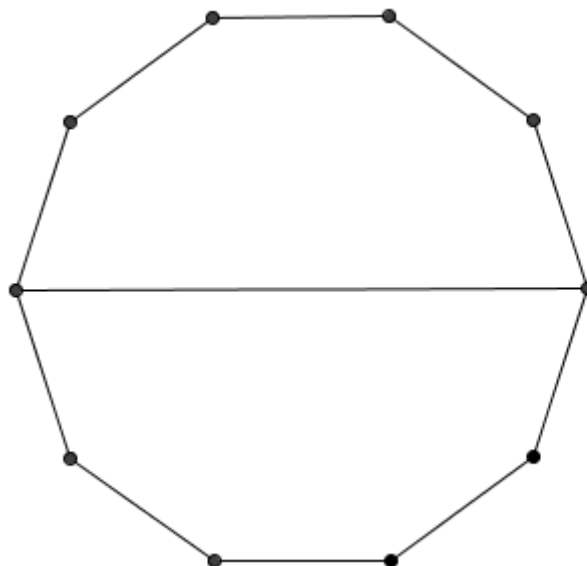
Shape	a	b	h	$A = h(\frac{a+b}{2})$
1	y_1	y_2		
2				
3				
4				
5				
6		y_7		

Total area =

Simplify this formula:

Can you generalise this formula for dividing a shape into any number of trapeziums of equal h where the last side is of length y_n ?

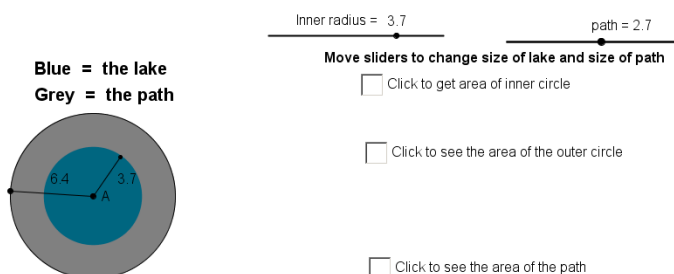
11. Use measurement and the Trapezoidal Rule to find the approximate area of the following shape in cm^2 .



12. Use measurement and the Trapezoidal Rule to find the approximate area of the following shape in cm^2 .

Student Activity: To investigate the area of a circular pond and surrounding path

Use in connection with the interactive file, 'Circle Pond', on the Student's CD.



All measurements are in metres

- Find the area of a circle of radius 3.7 cm. Explain how you got your answer.

- A circular pond of radius 5 m is surrounded by a path of width 1 m.
 - What is the area of the pond?

- What is the area of the pond plus the path?

- What is the area of the path?

- d. Given that it costs €50 per m^2 to insert the path, how much will this path cost?

- e. If the width of the path was increased to 2 m, what effect would this have on the area of the pond plus path and consequently on the path? Explain your answer.

3. A circular garden of area $49\pi \text{ m}^2$ consists of a pond surrounded by a path of width 1 m. Find the area of the pond and the area of the path.

4. A circular garden consists of a pond of area $36\pi \text{ m}^2$ surrounded by a path of width 1m. Find the area of the pond and the area of the path.

5. A circular garden of area 78.5 m^2 consists of a pond surrounded by a path of width 1 m. Find the area of the pond and the area of the path.

6. A circular picture of area 452.16 cm^2 is surrounded by a frame of width 2 cm. Find the area of wall that this framed picture will cover.

7. When cooking a pizza of diameter 50 cm the chef overcooked it and exactly 2 cm around the circumference was inedible, what area of pizza was wasted? What percentage of the pizza was wasted?

8. A family want to extend their circular swimming pool through extending the radius by 100 cm. The current radius of the swimming pool is 6 m. Find the new area of their pool. By what percent did their swimming pool increase in size?

9. A family decides to reduce their circular flower bed whose radius is 4 m by cementing a border of width 2 m around the bed. What percentage of the bed will be lost?

10. When rolling out dough for a pizza that was originally going to have a radius of 20 cm, the chef decides to extend the radius of the circular base by 5 cm.

- a. What will the percentage increase in size of the pizza be?

- b. Calculate the area of the base of a rectangular box that will be required to hold the new pizza?

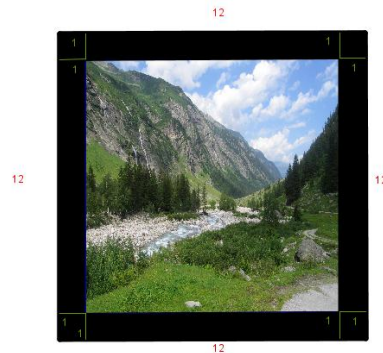
- c. Given that the height of the box will be 8 cm, draw a labelled net of the required box.

- d. Find the volume of the box.

- e. Find the total surface area of the box.

Student Activity: To investigate a frame on a picture

Use in connection with the interactive file, 'Rectangular Picture Frame', on the Student's CD.



- 1) Gary has a framed square picture, where each outer edge of the frame measures 12 cm as above.

- a. What area of wall surface does this framed picture cover?

- b. If the frame on this picture is 1 cm wide all around,

- i. What is the width and length of the actual picture inside the frame?

1.

2.

3.

- ii. What is the area of the picture?

- c. What is the area of the black frame around this picture?

- d. What wall surface would be covered if we increased the frame to 2 cm around the picture as used in part **b** of this question?

- e. What wall surface would be covered by the framed picture if we increased each side length of the actual picture inside the frame to 15 cm and kept the frame width as 1 cm?

2) Complete the following table:

Length of the side of a square frame	Width of the frame	Length of side of the square picture inside the frame	Area of the picture inside the frame
10	1		
10	2		
10	3		
10	4		

- a) Why is it not possible to frame a square picture inside a square frame of side length 10 and have a frame of width 5 cm?

- b) If x is the length of the side of the square frame and y is the width of the frame surrounding a picture inside the frame, what is the area of the picture?

- 3) What area of wall would a picture of length 30 cm and width 20 cm cover if it is enclosed by a frame of uniform width 5 cm?

- 4) A gardener has a square garden of area 144m^2 and decides to put a path of uniform width of 1m around the edge of the garden and leave the rest as a lawn. Find the area of the lawn.

- 5) A hotel decides to put a swimming pool in one of its rooms. The length of the room is 28m and the width is 25m. The pool will need a path of 2m surrounding it at all times.

- a) Find the area of the biggest possible pool.

- b) Find the area of the path and the cost of the path if it costs €50 per m^2 to insert.

- c) What will the perimeter of the pool be?

- 6) A room has width 4m, length 5m and height 3m. The owner decides that he is going to add insulation material of depth 10 cm to each wall.

- a) What will the new area of the floor of the room be?

- b) How much floor space will the owner lose?

- c) What will the new area of each wall of the room be?

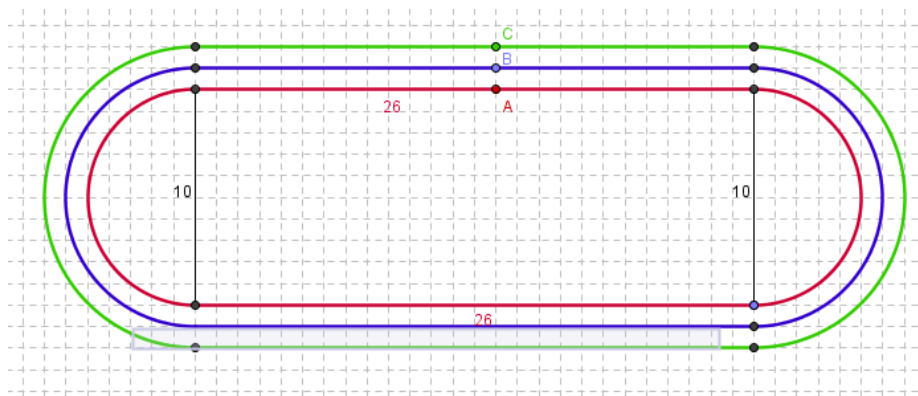
- d) What will the new volume of the room be?

- e) How much space (volume) will the owner have lost?

- 7) A farmer has a vegetable plot of length 5m and width 6m in a field and decides to increase the plot in size by adding a length of 1m to each side. What will the new area of his plot be and by what percentage will the area of his garden have increased?

Student Activity: To investigate a rectangular race track with circular ends

Use in connection with the interactive file, 'Race Track', on the Student's CD.



Note: In the diagram above the straight sections of each track is 26m, the diameter of the inner track is 10m and the distance between each track and the next track is 1m.

1. Given that the diameter of the inside semicircle is 10m, find
 - a. The diameter of the second semicircle.

 - b. The diameter of the third semicircle.

 - c. The circumference of the third semicircle. Show your calculations.

2. If runner A starts at point A on the diagram and follows the inner line on the track in the diagram, what distance will this runner need to run in order to complete one circuit of the track? Show your calculations.

3. What impact would a different starting point have on the distance runner A had to run, provided he followed the same track and arrived back where he started?

4. If runner B follows the middle track in the diagram:
- What will the radius of the semicircular section of runner B's track be if the distance between the tracks is 1m?

- What distance will runner B have to run in order to complete one circuit of the track, if this runner starts at point B in the diagram? Show your calculations.

- How far would runner B's starting point need to be moved forward in order to make it a fair race, such that runners A and B would have the same distance to travel? Explain your answer.

5. If runner C follows the outer track in the diagram:

- What will the radius of the semicircular section of runner C's track be if the distance between the tracks is 1m? Show your calculations.

- What distance will runner C need to run in order to complete one circuit of the track, if this runner starts at point C in the diagram? Show your calculations.

- How far would runner C's starting point need to be moved forward in order to make it a fair race, such that runners A and C would have the same distance to travel? Explain your answer.

6. Assuming there is a runner D, who runs on a track 1m out from runner C:
- What will the radius of the semicircular section of runner D's track be given the distance between the tracks is 1m? Show your calculations.

- What distance will runner D need to run in order to complete one circuit of the track? Show your calculations.

- How far would runner D's starting point need to be moved forward in order to make it a fair race, such that runners A and D would have the same distance to travel? Explain your answer.

7. What effect would increasing the diameter of the inner circular track by 1m have on the distance runners A, B and C would have to run in order to do a complete circuit of their tracks?

8. What effect would increasing the straight section of the inner track by 1m have on the distance runners A, B and C would have to run in order to do a complete circuit of their tracks?

9. Would doubling the diameter of the inner circular track and doubling the straight section of the track double the length of the track for runner A, who runs on the inner track? Explain your answer.

10. A gardener has a flower bed in the shape of a rectangle with a semi circle at each end. The dimensions of the rectangle are 3m by 4m and the diameter of the semicircle is 4m.

a. Draw a scaled diagram of the flower bed using the scale 1m=2cm.

b. What is the area of the flower bed?

c. The gardener decides to add a 1m path around the outside of the garden. What will the inside length of this path be?

d. What will the area of the garden plus path be?

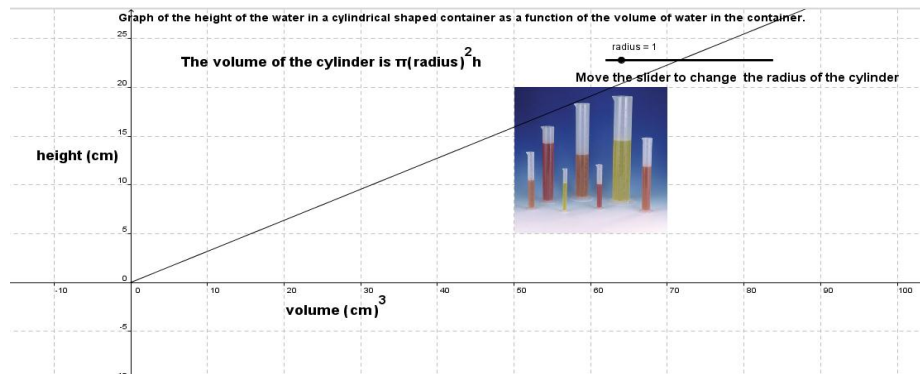
e. What will the area of the path be?

f. What will be the minimum dimensions of the garden be in order to accommodate the flowerbed and path? Represent this in a diagram.

g. The gardener decides to raise the flower bed by a uniform height of 50cm. What volume of soil will be required to do this?

Student Activity: To investigate the rate of change of height with respect to volume

Use in connection with the interactive file, 'Height with respect to volume', on the student's CD.

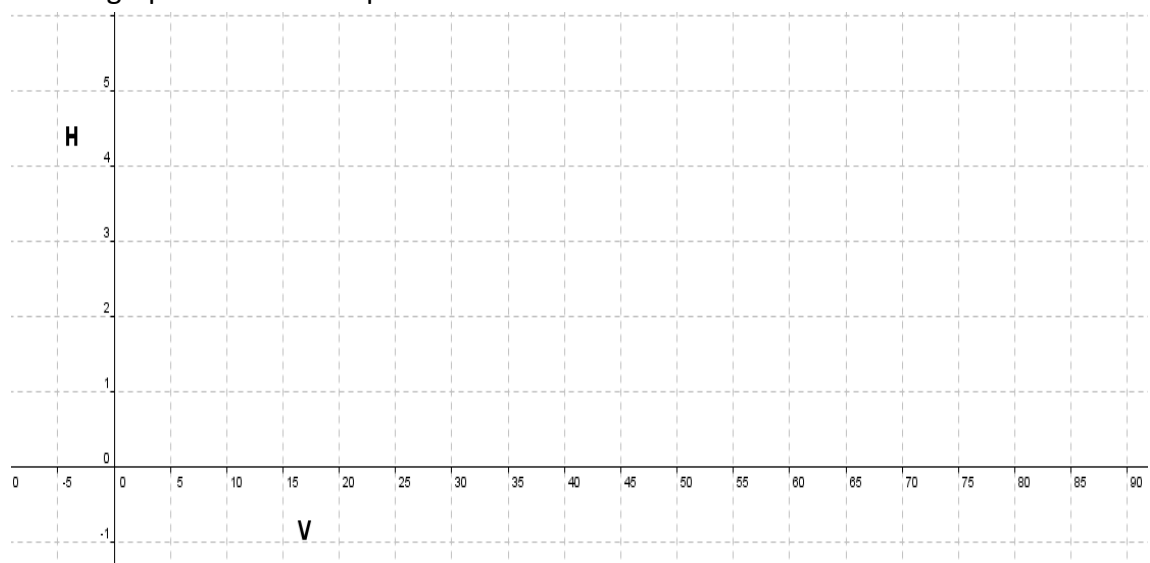


1.

- a. Using a calculator complete the following table for a container of radius $r = 2$ cm and height h :

Volume (cm) ³	$h = \frac{\text{volume}}{\pi(r)^2} \text{ (cm)}$
10	
20	
30	
40	
50	

- b. Draw a graph of the data represented in the table.



- c. Did your data form a linear, quadratic or exponential graph? Explain your answer.

- d. Is the height of this cylinder proportional to its volume? Explain your answer.

- e. What does the slope mean in the context of this problem?

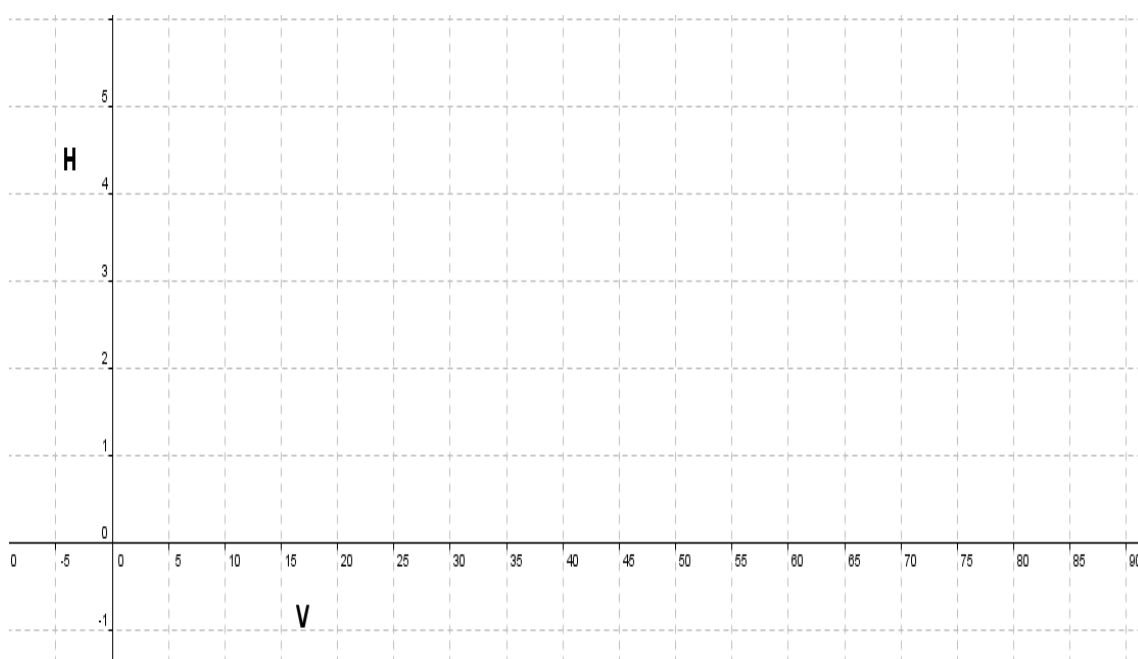
- f. Is the rate of change of the height with respect to volume increasing, decreasing, or constant for this problem? Explain your answer.

2.

- a. Using a calculator, complete the following table for a container of radius $r = 3$ cm and height h :

Volume (cm) ³	$h = \frac{\text{volume}}{\pi(r)^2}$ (cm)
10	
20	
30	
40	
50	

- b. Draw a graph of the data represented in the table.



- c. Did your data form a linear, quadratic or exponential graph? Explain your answer.

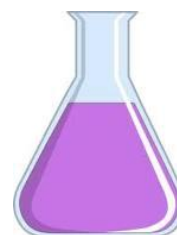
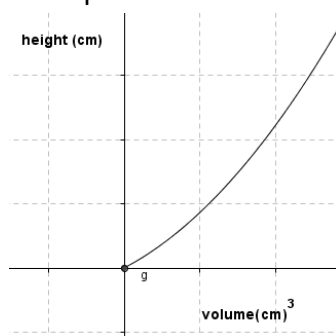
- d. Is the height of this container proportional to its volume? Explain your answer.

- e. What does the slope mean in the context of this problem?

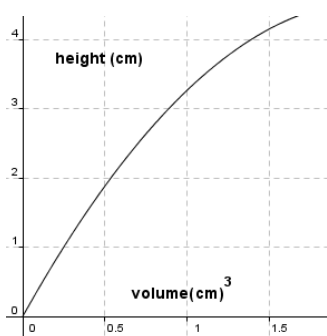
- f. Is the rate of change of the height with respect to volume increasing, decreasing, or constant for this problem? Explain your answer.

3. Using the two graphs you have drawn and the interactive file, do you agree with the statement that “The greater the radius of the container the slower the rate of change of the height (cm) with respect to the volume (cm)³.” Explain your reasoning.

4. Following the logic obtained in the above questions can you explain why, if the radius of a container is getting smaller as in the flask opposite, the graph of volume (cm)³ vs. height (cm) will be shaped as follows:



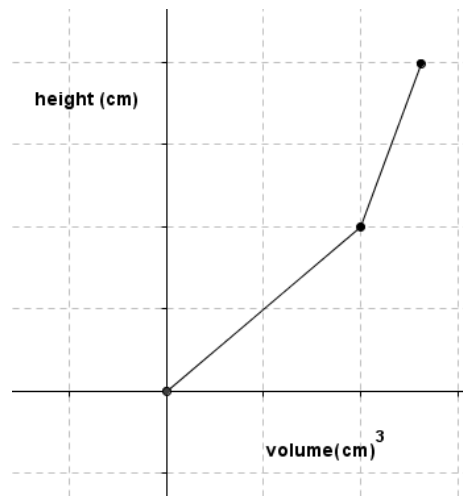
5. Following the logic obtained in the above questions can you explain why if the radius of a container is getting larger as in the container opposite, the graph of volume (cm)³ vs. height (cm) will be shaped as follows:



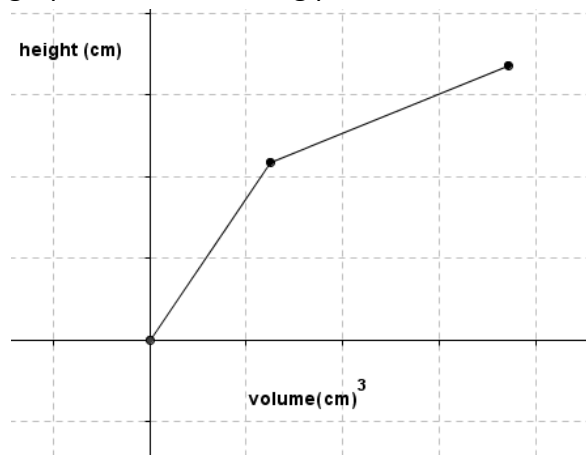
6. Draw a rough sketch of the graph of the volume vs. the height of the following container. Note water cannot enter the handles.



7. Draw a rough sketch of the shape of the container represented by the following graph as water is being poured into it.



8. Draw a rough sketch of the shape of the container represented by the following graph as water is being poured into it.



Student Activity: To investigate the square root of two and the square root of three

Use in connection with the interactive file, 'Square root 2' and 'Square root 3', on the Student's CD.

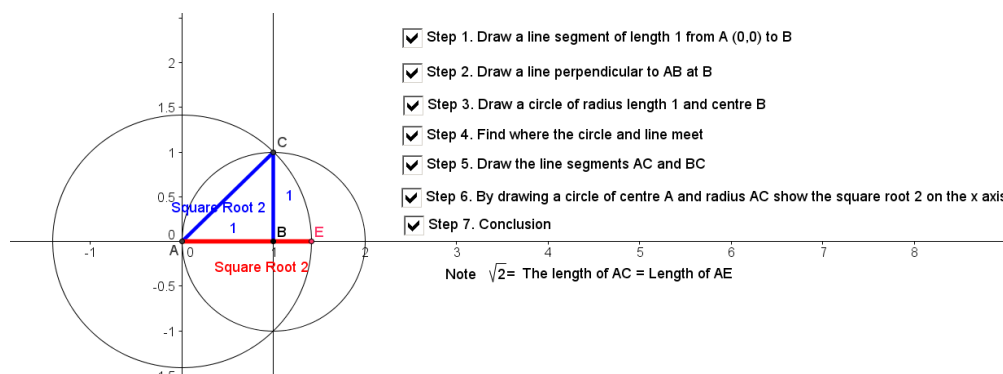


Fig. 1

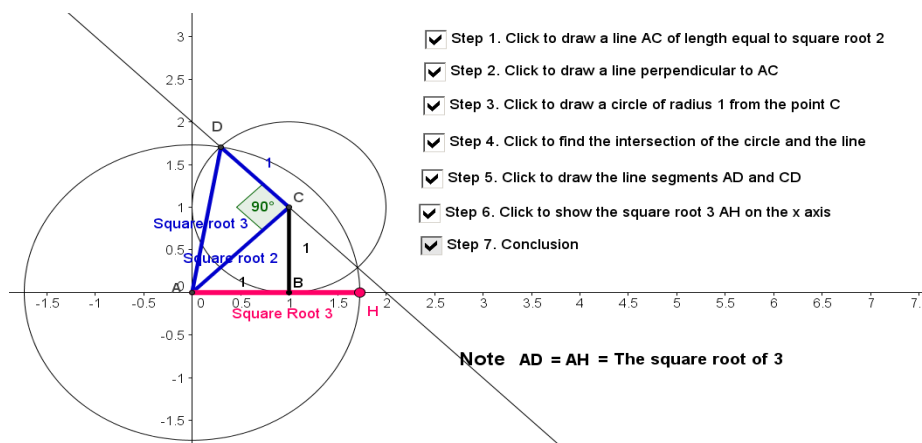


Fig.2

1. What is meant by the square root of a number?

2. Given that $\sqrt{a}$ is the positive square root of a , answer the following:

a. $\sqrt{9} =$

b. $\sqrt{16} =$

c. $\sqrt{2500} =$

3. What is $\sqrt{1}$? Represent this length on a line using centimetres as the units.

4. What is $\sqrt{4}$? Represent this length on a line using centimetres as the units.

5. Estimate $\sqrt{2}$.

6. In the interactive file "Square root two", what type of triangle is formed by the points ABC and can Pythagoras' theorem be applied to this triangle?

7. What are the lengths of side $|AB|$ and $|BC|$?

8. Using Pythagoras' theorem, what is the length of side $|AC|$?

9. What line segment is $|AC|$ equal to on the number line?

10. With the help of the interactive file, construct the square root of two below and show its position on the number line.

11. In the interactive file "Square root three", what type of triangle is formed by the points ACD and can Pythagoras' theorem be applied to this triangle?

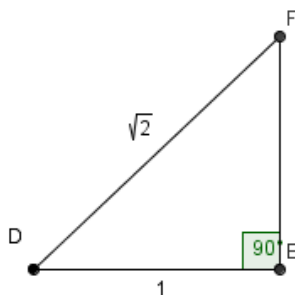
12. In the interactive file "Square root three", what are the lengths of $|AC|$ and $|CD|$?

13. Using Pythagoras' theorem, what is the length of side $|AD|$?

14. What line segment is $|AD|$ equal to on the number line? _____

15. With the help of the interactive file, construct the square root of 3 below and show its position on the number line.

16. Using $|DE|$ as one unit, find the length of the line segment $|EF|$ in the following diagram and the measure of the angles $|DFE|$ and $|EFD|$.



17. Find the length and width of an A4 sheet of paper.

18. Find the length and width of an A3 sheet of paper.

19. What do you notice about the ratio between the length of the A3 sheet of paper and the A4 sheet of paper?

20. What do you notice about the ratio between the width of the A3 sheet of paper and the A4 sheet of paper?

21. Draw conclusions between the ratio of an A3 sheet of paper in comparison to an A4 sheet of paper correct to 3 decimal places and $\sqrt{2}$.

22. Check if this ratio holds true for other paper sizes?
©<http://www.papersizes.org/a-paper-sizes.htm>

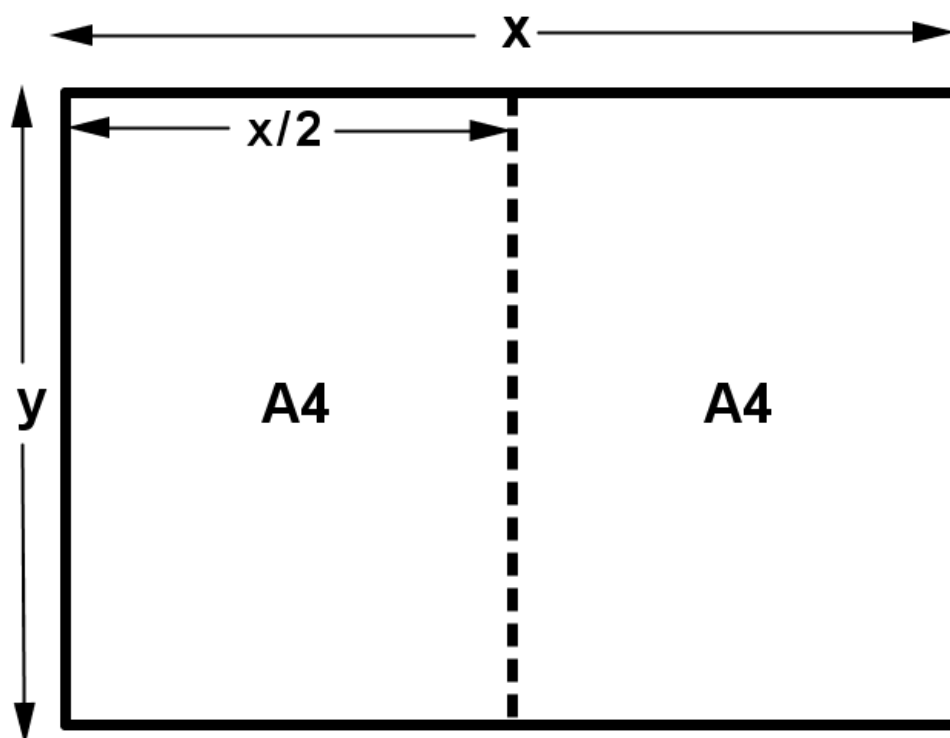
Size	Height x Width (mm)
4A0	2378 x 1682 mm
2A0	1682 x 1189 mm
A0	1189 x 841 mm
A1	841 x 594 mm
A2	594 x 420 mm
A3	420 x 297 mm
A4	297 x 210 mm
A5	210 x 148 mm
A6	148 x 105 mm
A7	105 x 74 mm
A8	74 x 52 mm
A9	52 x 37 mm
A10	37 x 26 mm

Challenge

23. Given rectangle $AEFC$ below represents an A3 sheet of paper :
- What is the ratio of length (longest side) to width for rectangle $AEFD$ in terms of x and y ?

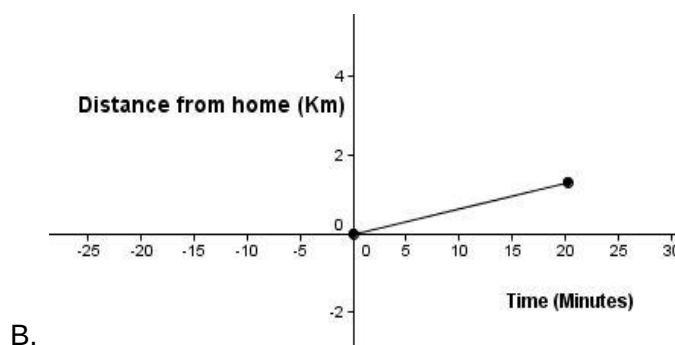
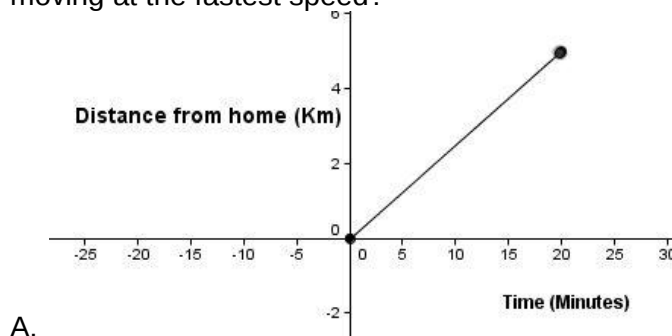
Rectangle $AEFC$ is folded in two to give 2 rectangles as shown $ABDC$ and $BEFC$. What is the ratio of length (longest side) to width for rectangle $ABCD$?

- Write an expression for the relationship of x to y which will preserve the same ratio of length to width for rectangle $ABDC$ as the ratio of length to width for $AEFC$.



Relations without Formulae Quiz

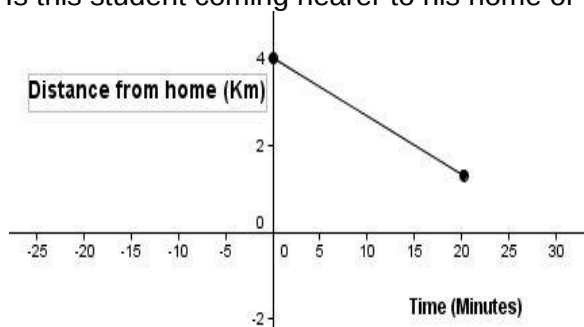
1. Given that these graphs represent someone cycling, in which graph is the cyclist moving at the fastest speed?



A. A.

B. B.

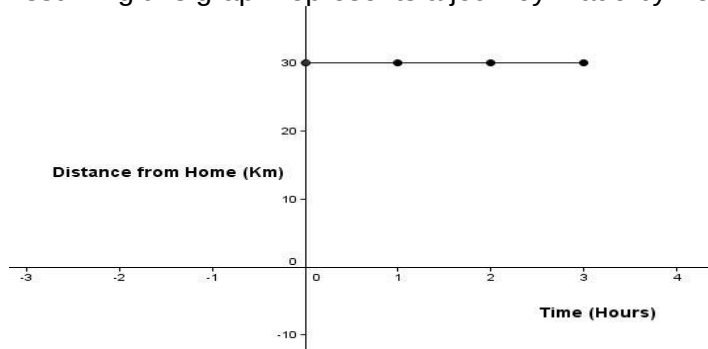
2. Is this student coming nearer to his home or moving away from his home?



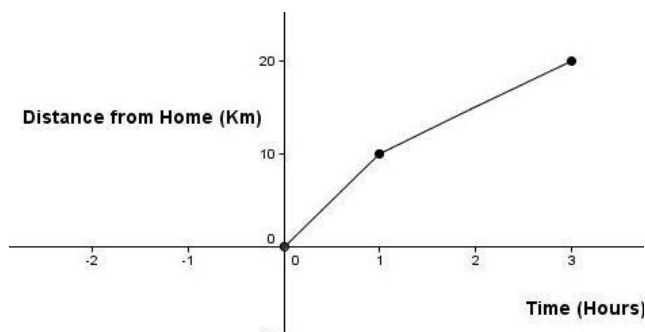
A. He is coming nearer his home

B. He is moving away from his home

3. Assuming this graph represents a journey made by Kevin, which story is true?

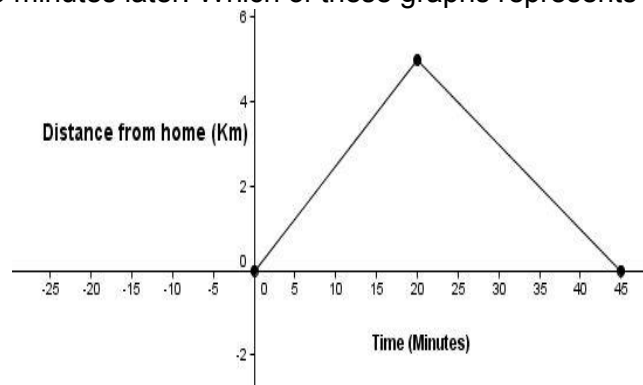


- A. Kevin started his journey 30 km from home and arrived home after 3 hours.
 - B. Kevin started his journey 30 km from home and did not move for the 3 hours shown in the graph.
 - C. Kevin started his journey and travelled 3 km in 30 minutes.
4. Assuming this graph represents a journey made by Freda, which story is represented in the graph?

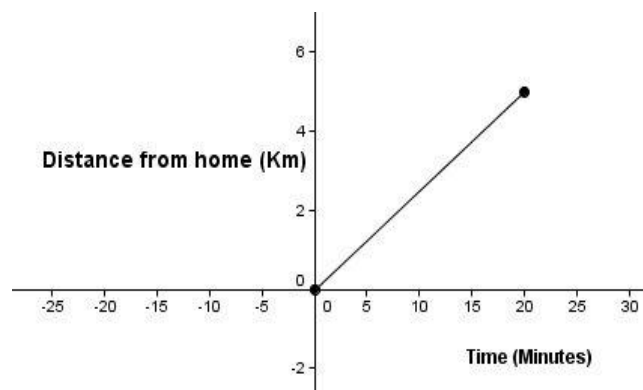


- A. In the first hour Freda travels at 10 km per hour and in the next hour she travels 20km.
- B. In the first hour Freda travels at 10 km per hour and in the next 2 hours she travels 10 km per hour.
- C. In the first hour Freda travels at 10 km per hour and in the next 2 hours she travels another 10km.
- D. In the first hour Freda travels at 10 km per hour and in the next hour she travels 10 km.

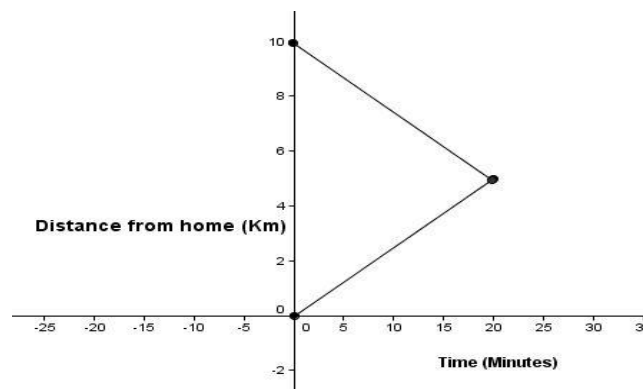
5. Fergus is a runner. He runs from his house at a steady speed until he gets to a marker, which is a distance of 5 kilometres from home, taking 20 minutes to do it. He immediately turns for home and runs at a slower, steady speed, arriving back a further 25 minutes later. Which of these graphs represents Fergus's run?



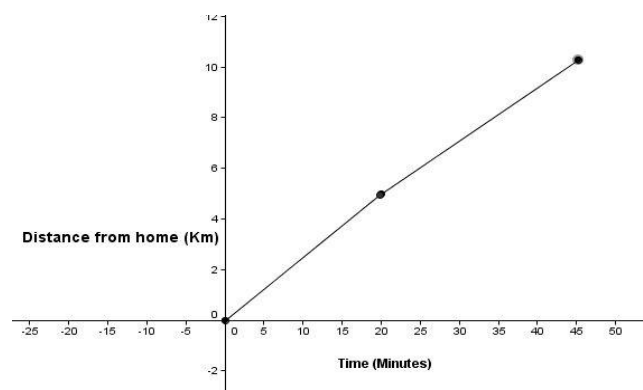
A.



B.

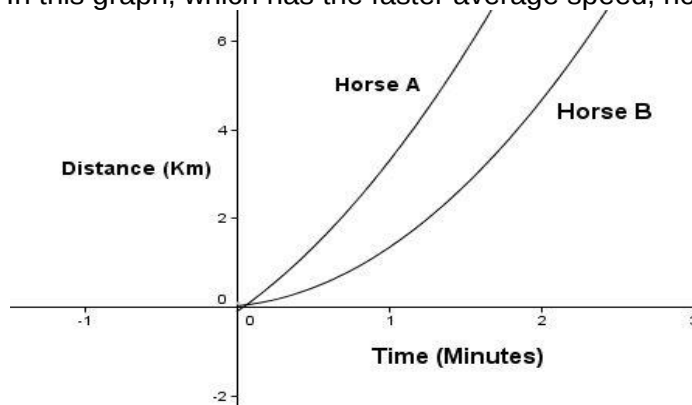


C.



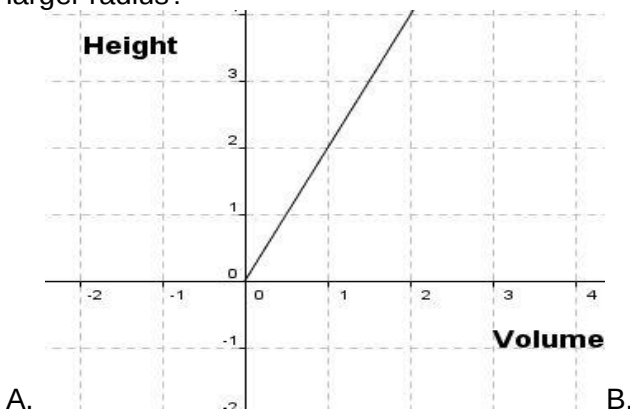
D.

6. In this graph, which has the faster average speed, horse A or horse B?

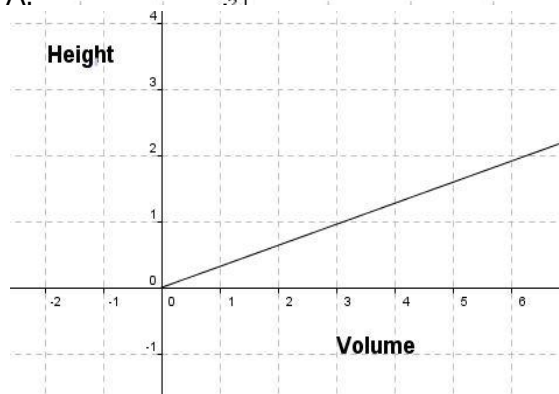


- A. Horse A
B. Horse B

7. Given that the following diagrams represents volume vs. height of a cylindrical shaped container, which of the following diagrams will represent the cylinder with the larger radius?

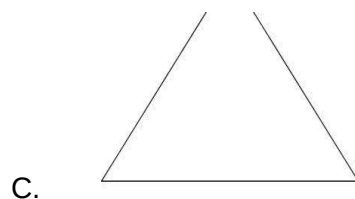
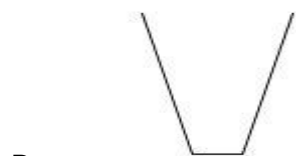
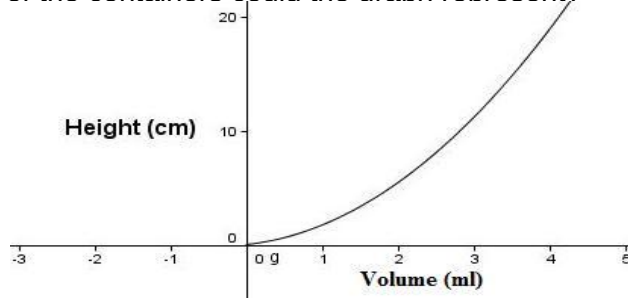


- A. B.

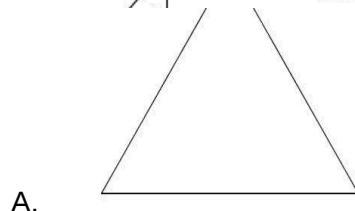
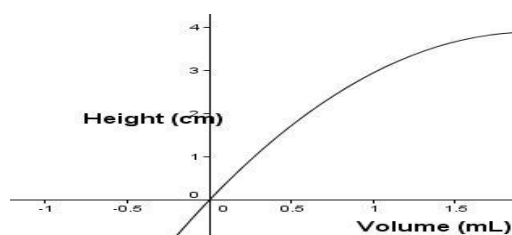


- A. A.
B. B.

8. This graph represents the volume of liquid in a container vs. the height of the liquid. The diagrams below are 2-dimensional representations of possible containers. Which of the containers could the graph represent?



9. This graph represents the volume of liquid in a container vs. the height of the liquid. The diagrams below are 2-dimensional representations of possible containers. Which of the containers could the graph represent?



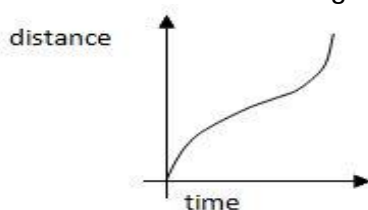


B.



C.

10. Match this time vs. distance graph with the correct statement.



- A. Moving at steady, moderate pace
- B. Moving at a very fast pace, then moving gradually to a slower pace
- C. Moving at a fast, steady pace
- D. Moving fast, then slowing slightly, then going faster again

Student Activity: To investigate how to calculate $x = \log_n m$

Use in connection with the interactive file, 'Changing log bases', on the Student's CD.



$$\log_2(8) = \frac{\log_{10} 8}{\log_{10} 2} = \frac{0.9031}{0.301} = 3$$

$$2^3 = 8$$

1.

- a. Using the format used in the interactive file, write $m = \log_3 81$ as $m = \frac{\log_{10} x}{\log_{10} y}$.

- b. Use a calculator to calculate $m = \log_3 81$.

- c. Check your answer using the interactive file.

- d. What is 3^m , where m is the answer to part **b.** above?

2.

- a. Using the format used in the interactive file, write $r = \log_4 36$ as $\frac{\log_{10} x}{\log_{10} y}$.

- b. Use a calculator to calculate $r = \log_4 36$.

- c. Check your answer using the interactive file.

- d. What is 4^r , where r is the answer to part **b.** above?

3.

- a. Use your calculator to calculate $\log_{10} 1$.

- b. Prove that $\log_m 1 = 0$, for all m .

- c. Prove that $\log_{\frac{1}{5}} \frac{1}{x} = \log_5 x$.

- d. Show that $\log_r \left(\frac{1}{r^{-100}} \right) = 100$ for all values of r .

4. Given that $\log_3 x = a$ and $\log_3 4 = b$. Find $\log_4 x$ in terms of a and b .

5. If $\log_n m = b$:

- a. Write m as a power of n .

- b. Take the log of both sides of your answer in part a. Now prove that

$$B = \log_n m = \frac{\log_a m}{\log_a n}.$$

- c. Given that $\log_n m = \frac{\log_a m}{\log_a n}$, prove that $\log_n m = \frac{1}{\log_m n}$.

- d. Given that $\log_{10} 2 = b$, show that $\log_8 5 = \frac{1-b}{3b}$.

6. Prove that $\log_x y^4 z - \log_x y^2 z^2 = \log_x \frac{y^2}{z}$.

7. If $\log_4 y = 3$, find $\log_4 \left(\frac{1}{16} y \right)$.

Logs Quiz

1. If $\log_2 8 = x$, what is the value of x ?
 - A. 3
 - B. 2
 - C. 8
 - D. 4

2. Which of the following is equal in value to $\log_{10} 1$?
 - A. 1
 - B. 0
 - C. 10

3. Which of the following is equal in value to $\log_{10} 10$?
 - A. 1
 - B. 0
 - C. 10

4. Which of the following is equivalent to $\log_a b + \log_a c$?
 - A. $\log_a (b+c)$
 - B. $\log_a (bc)$
 - C. $\log_{bc} (a)$

5. Which of the following is equal in value to $\log_3 9 + \log_3 4$?
 - A. $\log_3 13$
 - B. $\log_3 2.25$
 - C. $\log_3 36$

6. Which of the following is equal in value to $\log_2 16 + \log_2 8$?
 - A. 128
 - B. 7
 - C. 6

7. Which of the following is equivalent to $\log_a x - \log_a y$?
- A. $\log_a xy$
- B. $\log_a (x + y)$
- C. $\log_a \left(\frac{x}{y} \right)$
- D. $\log_a (x - y)$
8. If $\log_{16} 8 = y$, what is the value of y ?
- A. $\frac{3}{4}$ B. 16 C. 8 D. $\frac{1}{2}$
9. Which of the following is equal in value to $\log_2 16 - \log_2 4$?
- A. $\log_2 20$ B. $\log_2 12$ C. 2
10. Which of the following is equal in value to $\log_a x^y$?
- A. $\frac{\log_a x}{y}$
- B. $y \log_a x$
- C. $\log_a x + \log_a y$
11. Which of the following is equivalent to $\log_4 x^3$?
- A. $4 \log_3 x$
- B. 12?
- C. $3 \log_4 x$
12. If $\log_5 x^2 = 4$, what is the value of x ?
- A. 4 B. 20 C. 25 D. 10

13. Which of the following is equivalent to $\log_8 6$?

A. $\frac{\log_{10} 8}{\log_{10} 6}$

B. $\log_{10} 6 - \log_{10} 8$

C. $\frac{\log_{10} 6}{\log_{10} 8}$

14. If $\log_{\frac{1}{2}} x = 8$, what is the value of x ?

A. $\frac{1}{512}$

B. 512

C. 64

15. Which of the following is equivalent to $\log_2 64 - \log_2 8$?

A. $\log_2 8$

B. 8

C. $\log_2 512$

16. Which of the following is equivalent to $\frac{\log_{10} 81}{\log_{10} 9}$?

A. 2

B. 90

C. 9

17. Which of the following is the solution to the equation $\log_x 32 + \log_x 4 = 7$?

A. 128

B. 36

C. 2

18. If $(64, 3)$ lies on the graph of $y = \log_b x$, what is the value of b ?

A. 4

B. 64

C. 32

19. If $\log_4(\log_9 x) = \frac{1}{2}$, find the value of x .

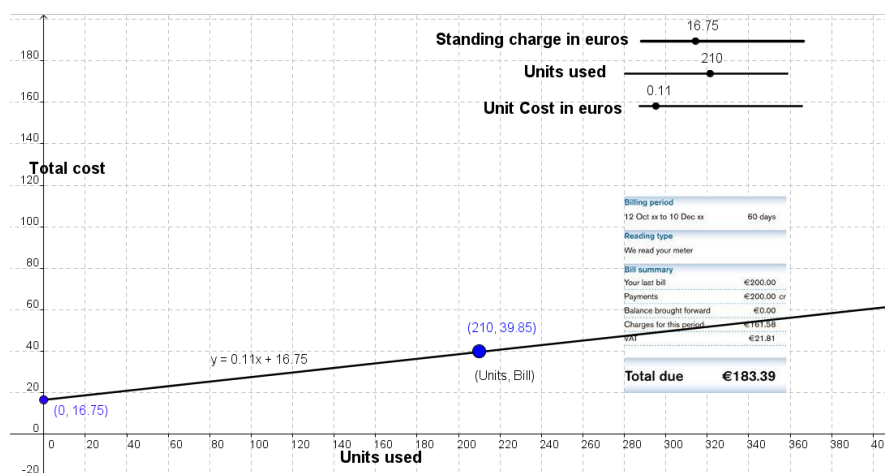
A. 18

B. 72

C. 81

Student Activity: To investigate an ESB bill

Use in connection with the interactive file, 'ESB Bill', on the Student's CD.



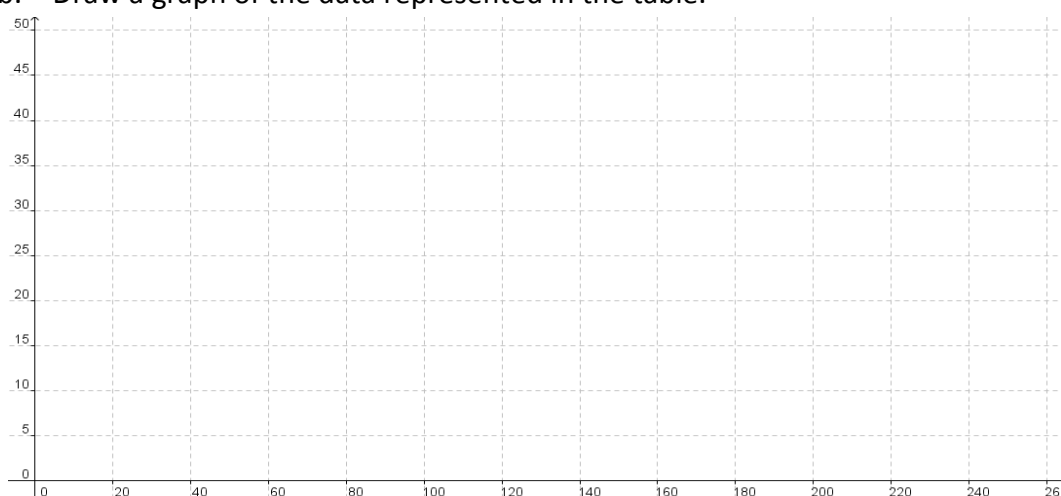
- What are the 2 main costs that contribute to your ESB bill?

2.

- Complete the following table to establish a household's ESB's costs, if the standing charge is €20 and the cost per unit is €0.10:

Units used	Cost of Units	Standing charge	Total Bill	Change in Total Bill
0				
50				
100				
150				
200				

- Draw a graph of the data represented in the table.



- c. What shape is the graph? Explain your answer.

- d. Where does this graph cut the y axis and what number does this relate to on the table?

- e. What is the rate of change of the graph?

- f. Find 2 points on the line and calculate the slope of the line.

- g. Write an algebraic formula to represent the cost of an ESB bill using the charges used in this question?

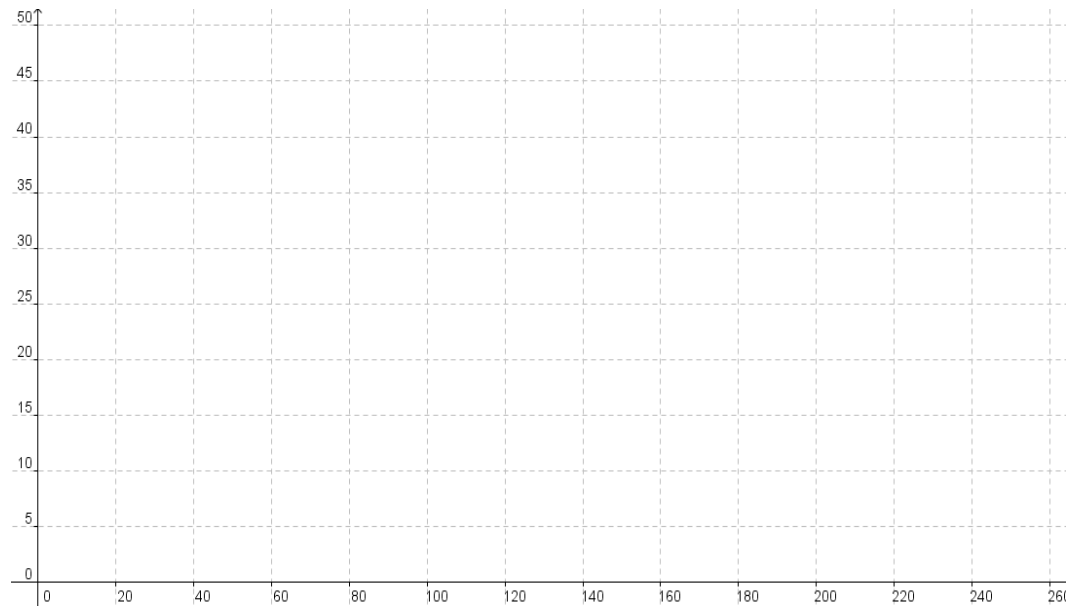
- h. Where does the slope appear i) in the table, ii) in the graph and iii) in the formula?

3.

- a. Complete the following table to establish a household's ESB's costs, if the standing charge is €15 and the cost per unit is €0.12:

Units used	Cost of Units	Standing charge	Total Bill	Change in Total Bill
0				
50				
100				
150				
200				

- b. Draw a graph of the data represented in the table.



- c. What shape is the graph? Explain your answer.

- d. Where does this graph cut the y axis and what number does this relate to on the table?

- e. What is the rate of change of the graph?

- f. Find 2 points on the line and calculate the slope of the line.

- g. Write an algebraic formula to represent the cost of an ESB bill using the charges used in this question?

- h. Where does the slope appear i) in the table, ii) in the graph and iii) in the formula?

4. In the interactive file, what determines:
 - a. Where the line cuts the y axis and where this is to be found in the equation of the line?

 - b. The slope of the line and where this is to be found in the equation of the line?

 - c. The point that represents the bill and where this is to be found in the equation of the line?

5. The current standing charge for your home is €12.25, the charge per unit is €0.11 and you used 250 units of electricity. What will your bill amount to assuming no extra charges? Show calculations and check your answer using the interactive file.

6. The current standing charge for your home is €22.25, the charge per unit is €0.10 and you used 260 units of electricity. What will your bill amount to assuming no extra charges? Show calculations and check your answer using the interactive file.

7. The current standing charge for your home is €32.25, the charge per unit is €0.11 and you used 256 units of electricity. What your bill will amount to assuming no extra charges? Show calculations and check your answer using the interactive file.

8. A family's last ESB bill was €44 and they used 200 units of electricity in that period. If the cost per unit was €0.12, what was the standing charge?

9. What 2 meter readings will be on your ESB bill?

10. How do you determine the number of units used by a household in a billing period?

11. If you are away on holidays for 2 months and used no electricity, will your bill be zero? Explain.

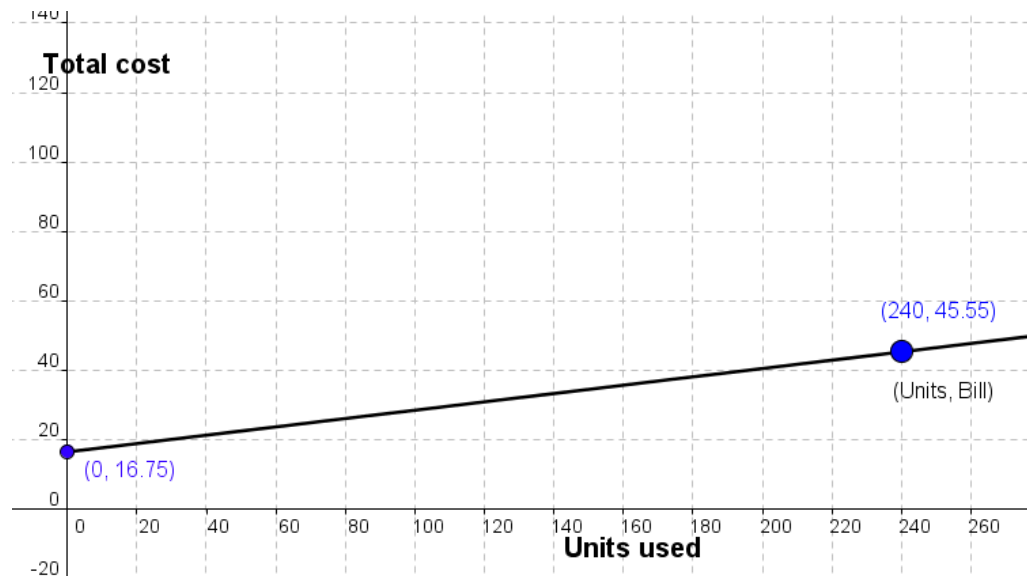
12. Which would be the better deal for a household that uses 300 units of electricity in a two month period? Explain your answer.

Deal A: Standing charge €30 and price per unit €0.10.

Deal B: Standing charge €10 and price per unit €0.20.

What type of household would each deal be more suited to? Explain your answer.

13.



- a. Using the diagram above, calculate the standing charge and the unit cost from the information shown.

- b. What is the equation of the line in the above diagram?

- c. Complete the table for the situation represented in the diagram above.

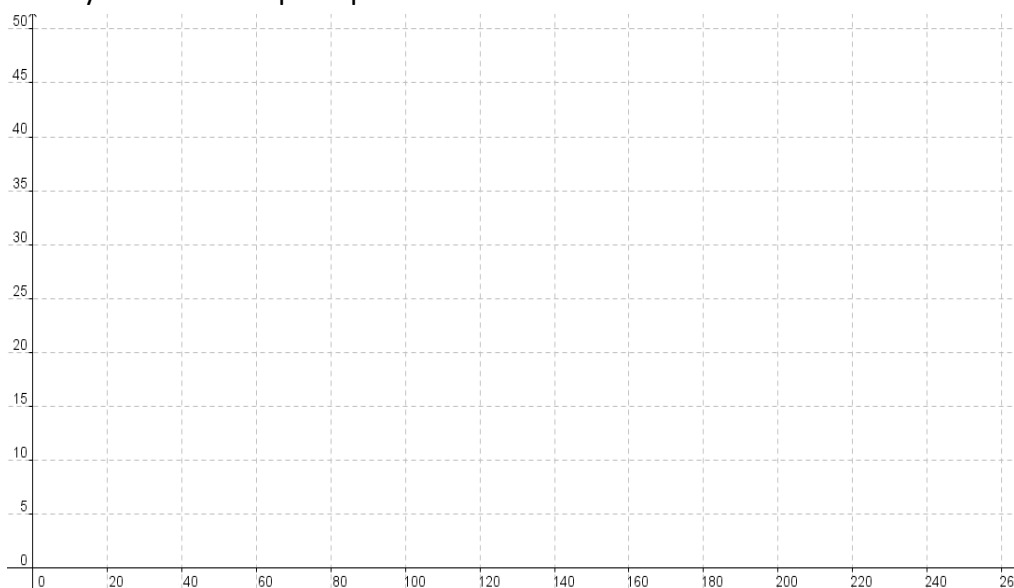
Units used	Cost of Units	Standing charge	Total Bill
0			
50			
100			
150			
200			

14.

- a. Complete the following table to represent the situation where the standing charge for electricity is €0 and the price per unit is €0.10.

Units used	Cost of Units	Standing charge	Total Bill
0			
50			
100			
150			
200			

- b. Draw a rough graph to represent the situation where the standing charge for electricity is €0 and the price per unit is €0.10.



- a. What is the formula that represents the situation where the standing charge for electricity is €0 and the price per unit is €0.10?

15. Jonathan forgot to pay his ESB bill for the last billing period and he used 200 units this period. The standing charge is €21 per 2 month period and the cost per unit is €0.10. If the bill for the 4 month period was €300, how many units did he use in the first 2 months assuming the charges remain the same? Show your calculations.

16. If the ESB abandoned its standing charge, but the amount of a household's bill did not increase, what 2 things could have happened?

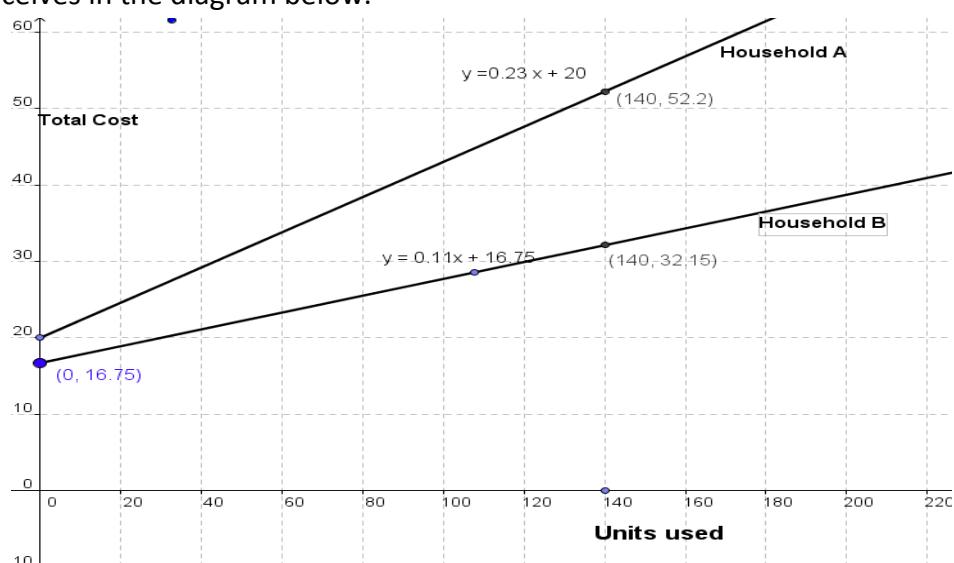
17. If you were told the number of units used vs. the total bill was represented by a linear graph $y=0.17x+10$. What does that tell you about the standing charge and cost per unit?

18. Describe what happens to the line in the interactive file if the cost of the standing charge for a household is increased.

19. Describe what happens to the line in the interactive file if the cost per unit is increased.

20.

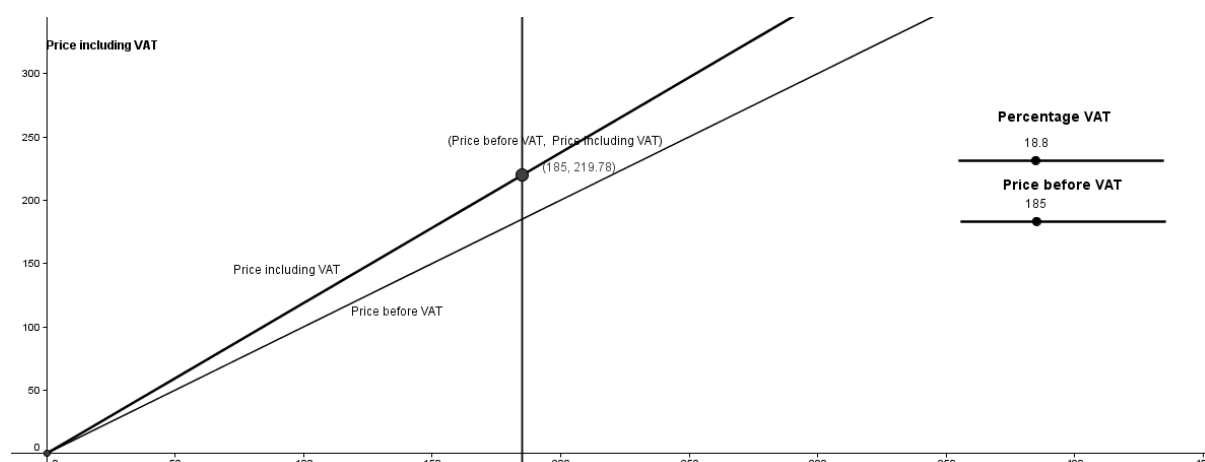
- a. Describe the cost of the bill Household A receives and the bill Household B receives in the diagram below.



- b. Write down and then graph a situation where a household has a higher standing charge than household A in the diagram above but ends up with a bill which is below Household A's bill after a certain number of units. Draw a rough sketch of this situation.

Student Activity: To investigate Value Added Tax (VAT)

Use in connection with the interactive file, 'VAT', on the Student's CD



1. Clothes cost €200 before VAT and VAT is charged at the rate of 20%. How much does the customer have to pay? Show your calculations.

2. A camera costs €238.95 and this price includes VAT at 18%. What is the price before VAT to the nearest cent? Show your calculations.

3. A set of DVD's costs €162.25 and this price includes VAT at 18%. How much VAT did the customer pay? Show your calculations.

4. VAT is increased from 12% to 21%.

- a. How much extra will someone have to pay for a television that cost €350 before VAT? Show your calculations.

- b. How much extra will a customer have to pay for a Nintendo that cost €130 before VAT?
Show your calculations.

5. Joe says “The more an item costs the greater the amount of VAT that has to be paid on it.” Is Joe correct? Discuss and use the interactive file to create an example to explain your answer.

6. Shoes cost €98.00 including VAT at 22.5%.

- a. What was the price of the shoes before VAT? Show your calculations.

- b. How much VAT was charged on these shoes? Show your calculations.

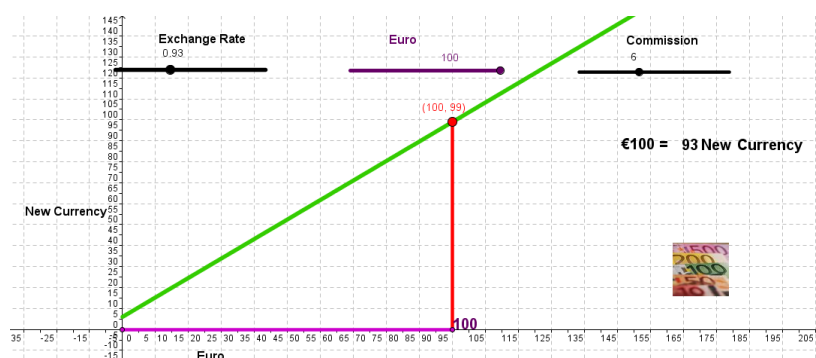
7. A family get an extension to their house costing €19000 before VAT. Given that the VAT rate on labour is 12% and the VAT rate on materials is 21%, how much will the extension cost if the labour costs amounted to €8500 and €10500 was spent on materials?

8. A magazine costs €2.60 and the rate of VAT on magazines is 15%. What is the cost of the magazine before VAT?

9. A tourist to Ireland buys sweaters costing €289.05 including VAT at 23%, but is told that he can get a refund of the amount of VAT paid on production of his receipt when leaving the country. How much of a refund is he entitled to?

Student Activity: To investigate exchange rates

Use in connection with the interactive file, 'Exchange Rate', on the Student's CD.



1. Convert €29 to USA dollars, if the exchange rate is €1=\$1.05 and the bank is charging commission of €4 per transaction. Show your calculations.

2. Which of the following is the better exchange rate if you were converting €120 to USA dollars and explain your choice?

Bank A charges commission at €2.50 per transaction and an exchange rate of €1 = \$1.02.

Bank B charges no commission and an exchange rate of €1 = \$1.10.

3. Bank C gives foreign exchange to students without commission. How much in euro, will Joseph who is a university student have to pay in Bank C in order to get \$125, provided the exchange rate on the day is €1=\$1.2? Show your calculations.

4. The exchange rate changes from €1 = \$1.2 to €1 = \$1.3. Will a customer who is converting €100 to dollars receive more or less dollars and by how much? Show your calculations.

5. Convert \$100 to euro given that the current exchange rate is €1 = \$1.24. Assume no commission is charged. Show your calculations.

6. A camera costs £150 in England and the exchange rate is €1= £0.82 plus the bank charges commission per transaction of €3. What will the camera cost in euro?

7. An American tourist is coming to Ireland and wishes to have €145 spending money while in Ireland. How many American dollars will they need to exchange to euro given \$1=€0.77? Show your calculations.

8. An Irish firm is selling goods in the UK that normally sell at €30 in Ireland. Ignoring the extra transport costs, what will they need to charge in sterling if the current exchange rate is €1=£0.90. Show your calculations.

9. Is it cheaper to holiday in America if the exchange rate is €1 = \$1.34 or €1 = \$0.89? Explain your reason.

10. An Irish firm, exporting goods to the UK, normally charge £5 per item for the goods in the UK. If the exchange rate changes from €1=£1.20 to €1=£0.80, is this better or worse for the Irish firm? Show your calculations and explain your reasoning.

11. Eve returns from Switzerland with 65 Swiss francs. She changes the francs back to euro, at a rate of €1=1.40 francs. How much does she get back? Show your calculations.

12. A certain bank advertises the sale of international drafts as follows:

Less than €600: 3.5% of the required amount subject to a minimum charge of €3 and maximum charge of €6.35 per draft

€600 and over: 0.75% of the required amount subject to a minimum charge of €6.35 and maximum charge of €25 per draft

Complete the following table:

Value of draft required	Charge
€50	
€100	
€400	
€600	
€700	
€30,000	
€40,000	

13. A tourist visiting Switzerland judges the cost of living in Switzerland in relation to Ireland on the price of a particular item. The exchange rate is €1=1.35 Swiss Francs (CHF). The item costs €50 in Ireland and costs 65 CHF in Switzerland. Should the tourist decide that Switzerland is more expensive or cheaper than Ireland? Show your calculation and explain your reasoning.

14. An item costs 4950 South African rand and the exchange rate between euro and the South African rand is one rand equals €0.11. How much would you expect to pay for this item in Ireland assuming the cost of the living in the two countries was the same? Show your calculations.

15. A bank charges €3551.36 for 8000 Brazilian Real. What is the exchange rate between euro and the Brazilian Real? Show your calculations.

16. A DVD is selling for €30.00 in Ireland and the same DVD is selling for \$35 in the USA. The exchange rate is \$1.00=€ 0.63. Is there a price difference between the two countries and if so state the percentage difference, correct to one decimal place, in the price in Ireland to that in the USA? Show your calculations.

Indices Quiz

1. Write $a^2 \times a^3$ in the form a^n .

A. a^5

B. a^6

2. Write $a^{-2} \times a^3$ in the form a^n .

A. a^{-6}

B. a^{-1}

C. a^1

D. a^{-5}

3. Write $\left(\frac{4}{7}\right)^{-1}$ as a rational number.

A. $-\frac{4}{7}$

B. $-\frac{7}{4}$

C. $\frac{7}{4}$

D. $\frac{4}{7}$

4. Simplify $2 \times 10^3 + 2 \times 10^2$.

A. 400000

B. 2200

C. 202

5. Evaluate $36^{\frac{1}{2}}$
- A. 18
- B. 3
- C. $\frac{1}{36}$
- D. 6
6. Write $a^{-2} \times a^{-3}$ in the form a^n .
- A. a^{-5}
- B. a^6
- C. a^5
- D. a^1
7. Write $2^{-2} \times 2^4 \times 8$ in the form 2^n .
- A. 2^{-24}
- B. 32^2
- C. 2^5
- D. $2^2 \times 8$
8. Which of these values are equal to 235^0 ?
- A. 235
- B. 0
- C. 1

9. Evaluate $81^{\frac{1}{4}}$.

A. 3

B. $\frac{1}{81}$

C. $\frac{1}{3}$

10. Write $(5^3)^2$ in the form 5^n .

A. 5^5

B. 5^6

C. 5^1

11. Is $2^2 \times 3^2 \times 5^2 = (2 \times 3 \times 5)^2$?

A. Yes

B. No

12. Which of the following is equal to $2 \times 10^{-3} \times 3 \times 10^6$?

A. 5×10^{-18}

B. 6×10^3

C. 6×10^{-3}

13. Write $3^5 \times 3^0$ in the form a^n .

A. 3^5

B. 3^6

C. 3^0

14. Write $125^{\frac{2}{3}}$ in the form 5^m .

A. 5^{-2}

B. 5^{-3}

C. 5^2

15. If $49^m = 7^{3-m}$. Find m.

A. 3

B. $\frac{1}{3}$

C. 1

16. Simplify $\frac{4x^2y^3}{xy}$.

A. $4xy$

B. 4

C. $4xy^2$

17. Simplify $(3m^{-2})^{-2}$.

A. $\frac{1}{3m^2}$

B. $\frac{1}{3m^4}$

C. $\frac{m^4}{9}$

D. $\frac{1}{3m^4}$

18. Write $5^{-\frac{2}{3}}$ as a root.

A. $\frac{1}{\sqrt[2]{5^3}}$

B. $\sqrt[3]{5^2}$

C. $\frac{1}{\sqrt[3]{5^2}}$

D. $\sqrt[2]{5^3}$

19. Write $\frac{1}{\sqrt[4]{x^3}}$ in the form x^n .

A. $x^{-\frac{3}{4}}$

B. $x^{\frac{3}{4}}$

C. $x^{\frac{4}{3}}$

D. $x^{\frac{4}{3}}$

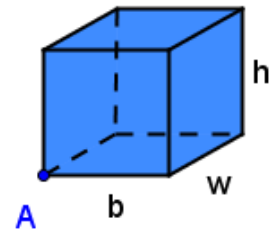
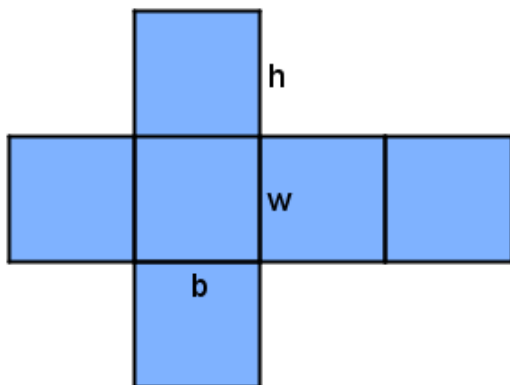
20. Simplify $\left(\frac{3x^5y^3}{2y^2}\right)^{-2}$.

A. $\frac{2}{3x^{10}y^2}$

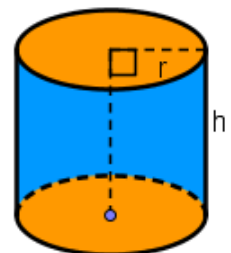
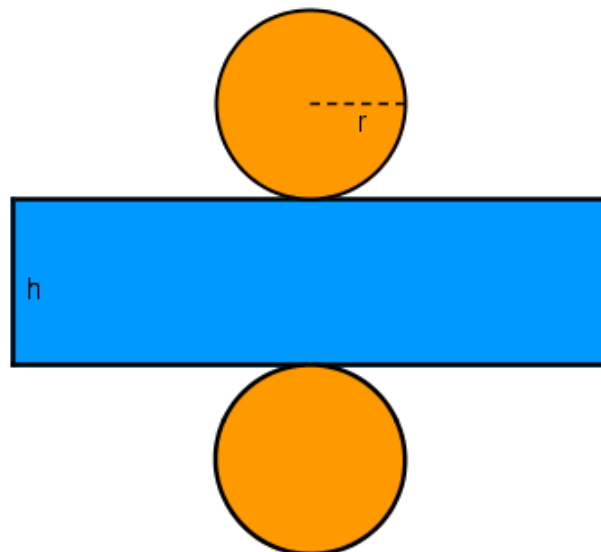
B. $\frac{4}{9x^{10}y^2}$

C. $\frac{9x^{10}y^2}{4}$

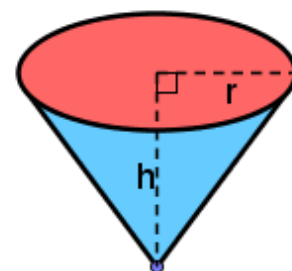
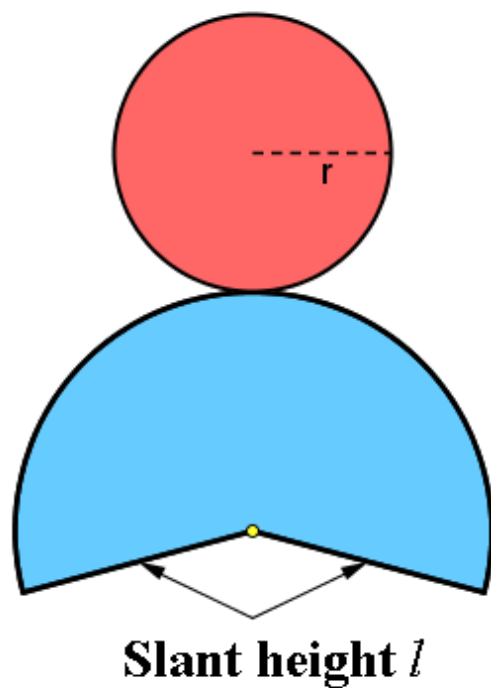
Net of a Cube



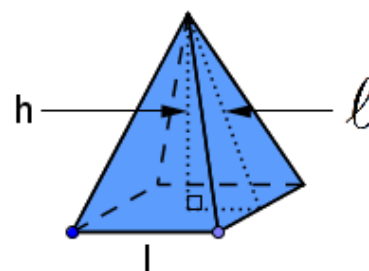
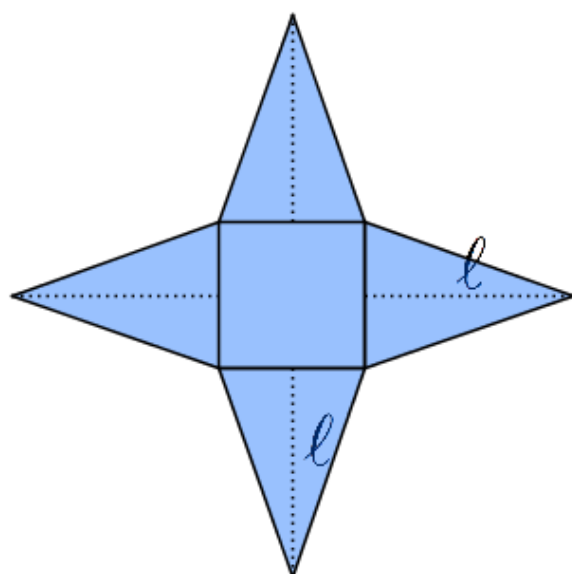
Net of a Cylinder



Net of a Cone



Net of a Pyramid



Net of a Octahedron

